

**MATH BOWL MATCH #6**  
**Round 1 (November 11, 2024)**

**1. Toss-Up**

On a blueprint, a rectangular room measures  $1\frac{3}{4}$  inches by  $2\frac{1}{4}$  inches. If the longer side of the actual room is 18 feet in length, how long is the shorter side of the actual room?

**Answer:** 14 feet

**Bonus**

On a blueprint, one foot is represented by a length of  $\frac{1}{24}$  of a foot. A room with an area of  $\frac{1}{6}$  of a square foot on the blueprint would have what actual area?

**Answer:** 96 square feet

**2. Toss-Up**

What is the value of  $x^4 - y^3$  when  $x = 3$  and  $y = 2$ ?

**Answer:** 73

**Bonus**

For what value of  $x$  does  $x^2 - 6x + 3$  have its minimum possible value?

**Answer:**  $[x = ] 3$

**3. Toss-Up**

Triangle ACF is congruent to triangle DGB. Therefore, the length of line segment AF must be the same as the length of what other segment?

**Answer:** DB [or BD]

**Bonus**

Triangle BED is similar to triangle COT. If BE = 5 inches, BD = 12 inches, and CT = 13 inches, what is the length of CO to the nearest tenth of an inch?

**Answer:** 5.4 [inches]

**4. Toss-Up**

A right angle is bisected, and the resulting angles are trisected. What is the measure of the smallest resulting angle?

**Answer:** 15 degrees

**Bonus**

What angle is formed by the hands of a clock at precisely 4:18?

**Answer:** 11 degrees

**5. Toss-Up**

The sum of three consecutive integers is 30. What is the smallest of the three integers?

**Answer:** 9

**Bonus**

An arithmetic sequence has fifth term  $2\frac{1}{3}$  and tenth term  $2\frac{3}{4}$ . What is the twelfth term of this sequence?

**Answer:**  $2\frac{11}{12}$

## 6. Toss-Up

The distance traveled by a ball rolling down a ramp is given by the equation  $s(t) = 3t^2$ , where  $t$  is the time in seconds after the ball is released and  $s(t)$  is the distance traveled in feet. How long will it take the ball to travel 27 feet?

**Answer:** 3 seconds

## Bonus

The distance traveled by a ball rolling down a ramp is given by the equation  $s(t) = 3t^2$ , where  $t$  is the time in seconds after the ball is released and  $s(t)$  is the distance traveled in feet. What is the average speed of a ball for the time between  $t = 1.5$  and  $t = 2.5$ ?

**Answer:** 12 feet per second

## 7. Toss-Up

If  $a * b = 3a - b - 1$ , what is the value of  $0 * 0$ ?

**Answer:** -1

## Bonus

If  $a * b = 3a - b - 1$ , solve the equation  $x * 2 = 2 * x$ .

**Answer:**  $[x = ] 2$

## 8. Toss-Up

Three circles are mutually externally tangent. A triangle is formed by connecting the centers of the three circles. If the radii of the circles are one foot, two feet, and three feet, what is the perimeter of the triangle?

**Answer:** 12 feet

## Bonus

Two circles are externally tangent. A line is tangent to both circles. If the radii of the circles are 3 feet and 4 feet, what is the distance on the line between the two points of tangency? Express your answer in simplest radical form.

**Answer:**  $4\sqrt{3}$  feet

## 9. Toss-Up

What is the distance between the points  $(3, 5)$  and  $(7, 2)$ ?

**Answer:** 5

## Bonus

What is the slope between the points  $(3, 5)$  and  $(7, 2)$ ?

**Answer:**  $\frac{-3}{4}$

## 10. Toss-Up

Five balls, numbered one through five, are placed in a jar. If two balls are drawn simultaneously and at random, what is the probability the sum of the numbers is 2?

**Answer:** 0

## Bonus

Five balls, numbered one through five, are placed in a jar. If two balls are drawn simultaneously and at random, what is the probability the sum of the numbers even?

**Answer:**  $\frac{2}{5}$

### 11. Toss-Up

Crazy Larry sets an explosive device and lays down 120 feet of fuse. He then lights the end. The fuse burns at the rate of 5 feet per second. Crazy Larry admires his handiwork for ten seconds, then realizes he needs to find a safer location before the explosion. How much time is left for Crazy Larry to take cover ?

**Answer:** 14 seconds

#### **Bonus**

Crazy Larry, after a lengthy recovery, sets an explosive device and lays down 300 feet of fuse. He believes the fuse will burn at a rate of 5 feet per second, but in reality, the fuse burns at 12 feet per second. How much sooner will the explosion occur than Crazy Larry would predict?

**Answer:** 35 seconds

### 12. Toss-Up

On a twenty-question multiple choice test, 0 points are awarded for an incorrect answer, 2 points are awarded if the answer is left blank, and 5 points are awarded for a correct answer. What is Sally's score if she left three answers blank and had four wrong answers?

**Answer:** 71

#### **Bonus**

On a twenty-question multiple choice test, 0 points are awarded for an incorrect answer, 2 points are awarded if the answer is left blank, and 5 points are awarded for a correct answer. If a score above 65 is considered passing, what is the smallest passing score that no one could possible get on this test?

**Answer:** 93

### 13. Toss-Up

What is the area of a circle with a diameter of 6 feet?

**Answer:**  $9\pi$  square feet

#### **Bonus**

A cone is intersected by a plane, forming a closed figure. If the plane is not parallel to the base of the cone, what is the term for the type of closed figure formed?

**Answer:** ellipse

### 14. Toss-Up

A rock band plays on Sunday, takes one day off, plays one day, takes two days off, plays one day, takes three days off, etc. This pattern continues, with the band taking one additional day off each time they play. On what day of the week will they play for the seventh time?

**Answer:** Saturday

#### **Bonus**

Bill and Bob each get two days off each seven-day week. Assuming these days are assigned randomly, what is the probability that Bill and Bob will both be off at least one day during a given week?

**Answer:**  $\frac{11}{21}$

### 15. Toss-Up

In how many ways can the letters in the word SODA be rearranged?

**Answer:** 24

#### **Bonus**

In how many ways can the letters in the word CAFFEINE be rearranged?

**Answer:** 10,080

**MATH BOWL MATCH #6**  
**Round 2 (November 11, 2024)**

**1. Toss-Up**

Find the sum of three tenths, seven hundredths, and five thousandths.

**Answer:**  $\frac{3}{8}$  [or 0.375]

**Bonus**

Find the difference between 8% and 0.7 as a fraction in simplest form.

**Answer:**  $\frac{31}{50}$

**2. Toss-Up**

Linda's mother is twice as old as Linda. Linda's sister is three years older than Linda. If Linda's sister is 18 years old, how old is Linda's mother?

**Answer:** 30

**Bonus**

Sam rides his bicycle downhill at twice the speed at which he rides uphill. If Sam takes 10 minutes to ride uphill to the park, how long will it take Sam to ride back from the park?

**Answer:** 5 minutes

**3. Toss-Up**

Chuck has a large supply of 10-cent stamps and 3-cent stamps. What is the largest value in cents that Chuck cannot make using some of his stamps?

**Answer:** 17 [cents]

**Bonus**

What is the largest integer that is less than its reciprocal?

**Answer:** -2

**4. Toss-Up**

At Washington High School, principals must serve exactly one four-year term. What is the largest number of principals that could serve during a ten-year period?

**Answer:** 4

**Bonus**

Every three-digit power of two contains what digit?

**Answer:** 2

**5. Toss-Up**

A triangle has sides of length 5 units and 12 units. If the third side is more than 13 units in length, then the triangle can be described by what adjective, based on the size of its largest angle?

**Answer:** obtuse

**Bonus**

What is the length of the hypotenuse of a right triangle with legs of length 15 inches and 20 inches?

**Answer:** 25 inches

**6. Toss-Up**

What is 70% of 200, increased by 10%?

**Answer:** 154

**Bonus**

After a 6% tax is applied, the cost of a car is \$15,344.56. What was the price before the tax was applied?

**Answer:** \$14,476.00

**7. Toss-Up**

What number is two less than three times itself?

**Answer:** 1

**Bonus**

If  $x$  divided by  $\frac{2}{3}$  is equal to  $x$  plus  $\frac{2}{3}$ , what is the value of  $x$ ? Express your answer as a mixed number in simplest form.

**Answer:**  $[x = ] 1\frac{1}{3}$

**8. Toss-Up**

What is the smallest prime number whose digits have a sum greater than 10?

**Answer:** 29

**Bonus**

What is the largest prime number to be one less than a perfect square?

**Answer:** 3

**9. Toss-Up**

What is the units digit of  $2008^{2009}$ ?

**Answer:** 8

**Bonus**

Let  $W(x)$  be the number of characters, counting hyphens but not spaces, in the written English form of  $x$ . Thus  $W(4) = 4$  (f-o-u-r) and  $W(41) = 9$  (f-o-r-t-y-hyphen-o-n-e). What is  $W(2008)$ ?

**Answer:** 16

**10. Toss-Up**

What is the sum of three cubed, three squared, and three factorial?

**Answer:** 42

**Bonus**

How many whole numbers less than 50 are prime?

**Answer:** 15

**11. Toss-Up**

Find the product of all whole numbers less than ten.

**Answer:** 0

**Bonus**

Find the sum of all whole numbers between one and forty, inclusive.

**Answer:** 820

**12. Toss-Up**

The area of a rectangle is 35 square meters. If the length is 7 meters, what is its perimeter?

**Answer:** 24 meters

**Bonus**

A right triangle has legs of length 56 centimeters and 90 centimeters. What is the length of the hypotenuse?

**Answer:** 106 centimeters

**13. Toss-Up**

The remainder when a two-digit number is divided by eight is five. What is the largest possible value for the number?

**Answer:** 93

**Bonus**

The remainder when a two-digit number is divided by three is one. The remainder when the number is divided by five is one, and the remainder when the number is divided by seven is five. What is the number?

**Answer:** 61

**14. Toss-Up**

A number has digits 4, 5, 3, 0, 2, 5, 6, 7, and  $x$ , in that order. If the number is to be divisible by nine, what must  $x$  be?

**Answer:** 4

**Bonus**

What is the sum of all prime factors of five million?

**Answer:** 7

**15. Toss-Up**

What is the term for the data value that occurs most often in a data set?

**Answer:** mode

**Bonus**

A data set contains the values, listed from least to greatest of 2, 3,  $x$ ,  $x$ ,  $y$ ,  $z$ ,  $z$ , and 8. If the mode of the data is 3, the median is 4, and the mean is 4.5, what is the value of  $z$ ?

**Answer:** 6



**MATH BOWL MATCH #6**  
**Solutions to Problem Solving (November 11, 2024)**

1. Bart tosses three nickels. Lisa tosses two dimes. What is the probability that Bart gets the same number of heads as Lisa?

Either both must get zero, one, or two heads. Lisa has a  $\frac{1}{4}$  chance of getting no heads, a  $\frac{1}{2}$  chance of getting one head, and a  $\frac{1}{4}$  chance of getting two heads. Bart has a  $\frac{1}{8}$  chance of getting no heads, a  $\frac{3}{8}$  chance of getting one head, a  $\frac{3}{8}$  chance of getting two heads, and a  $\frac{1}{8}$  chance of getting three heads. So  $P(\text{both get no heads}) = \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32}$ ,  $P(\text{both get 1 head}) = \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16}$ , and  $P(\text{both get 2 heads}) = \frac{1}{4} \cdot \frac{3}{8} = \frac{3}{32}$ . So the total probability is  $\frac{1}{32} + \frac{3}{16} + \frac{3}{32} = \frac{5}{16}$ .

2. In a list of nine numbers, the mean of the first five numbers is 10, and the mean of the last five numbers is 15. If the mean of all nine numbers is 12, what number is in the middle of the list? (Hint: the numbers are not necessarily in numerical order.)

Let  $X$  be the sum of the first four numbers, let  $Y$  be the sum of the last four numbers, and let the middle number be  $Z$ . Since the mean of the first five numbers is 10, their sum is  $5 \cdot 10 = 50$ , so  $X + Z = 50$ . Similarly, the mean of the last five numbers is 15, so their sum is  $5 \cdot 15 = 75$ , so  $Y + Z = 75$ . Adding these two equations,  $X + Y + Z + Z = 125$ .

Since the mean of all nine numbers is 12, their sum is  $9 \cdot 12 = 108$ , so  $X + Y + Z = 108$ . Substituting into  $(X + Y + Z) + Z = 125$ , we obtain  $108 + Z = 125$ , or  $Z = 17$ .

3. ABCD is a rectangle, with E the midpoint of BC and F the midpoint of CD. What is the area of  $\triangle AEF$  if the area of ABCD is 100 square units?

Let  $2x$  represent the length of AB (and CD) and let  $2y$  represent the length of BC (and DA). The area of  $\triangle AEF$  is the area of rectangle ABCD minus the areas of the three right triangles  $\triangle ABF$ ,  $\triangle FCE$ , and  $\triangle EDA$ . Now, the area of the ABCD is  $(2x)(2y) = 4xy$ . Since this is also 100, we know  $xy = 25$ . The area of  $\triangle ABF$  is  $\frac{1}{2}(2x)(y) = xy = 25$ , the area of  $\triangle FCE$  is  $\frac{1}{2}(x)(y) = \frac{1}{2}xy = 12.5$ , and the area of  $\triangle EDA$  is  $\frac{1}{2}(x)(2y) = xy = 25$ . So the area of  $\triangle AEF$  is  $100 - 25 - 12.5 - 25 = 37.5$ .

4. Twelve balls, numbered 1 through 12, are placed in a jar. Two balls are drawn simultaneously at random. What is the probability that the sum of these two numbers is odd?

Consider holding one ball in your left hand and the other in your right. For the sum of the two numbers to be odd, exactly one of them must be even, and the other must be odd. Whichever ball is in your left hand, there are six other balls that are of the other parity (even vs. odd), out of eleven balls from which to choose. Therefore, the probability is  $\frac{6}{11}$ .

5. Johnny never matches his socks; instead, he just throws them all in a drawer. The drawer currently holds eight pairs of socks, each a different color. In the dark, Johnny takes two socks at random. What is the probability that Johnny has a matching pair of socks?

Regardless of which sock Johnny picks first, only one sock of the 15 remaining will match, so the probability is  $\frac{1}{15}$ .