MATH BOWL MATCH #5
Round 1 (November 4, 2024)

1. Toss-Up
A cube is doubled in width, tripled in height, and quadrupled in length, forming a rectangular prism. By how many times larger in volume is the prism than was the cube?
Answer: 24

Bonus
A sphere varies in size over time, with its largest volume being twice its smallest volume. What is its largest diameter divided by its smallest diameter? Express your answer as a decimal to the nearest hundredth.
Answer: 1.26

2. Toss-Up
What is the value of $x^3 - x^2$ when $x = 6$?
Answer: 180

Bonus
What is the minimum value of the expression $x^2 - 4x + 7$?
Answer: 3

3. Toss-Up
A set of five data values has a mode of 6, a median of 7, and a mean of 8. If the largest data value is 12, what is the smallest data value?
Answer: 6

Bonus
In a set of eight data values, the mean of the three smallest values is 5, and the mean of the five largest values is 9. What is the mean of all eight data values?
Answer: 7.5

4. Toss-Up
If seven is multiplied by a number, the resulting product ends in the digit 1. What is the final digit of the original number?
Answer: 3

Bonus
When a two-digit number is divided by nine, the remainder is four. When the number is divided by eleven, the remainder is seven. What is the number?
Answer: 40

5. Toss-Up
What is the degree measure of an interior angle of a regular octagon?
Answer: 135 [degrees]

Bonus
What angle is formed by the hands of a clock at precisely 1:10?
Answer: 25 degrees

6. Toss-Up
The three medians of a triangle are drawn. The area of the smallest triangle formed is 10 square centimeters. What is the area of the original triangle?
Answer: 60 square centimeters

Bonus
Altitude AD is drawn in triangle ABC. The measure of angle B is 45 degrees. DC is one-half the measure of DB. If the length of AD is 4 inches, find the area of triangle ABC.
Answer: 12 square inches

7. Toss-Up
What number is four times its reciprocal?
Answer: 2

Bonus
The sum of a number and its reciprocal is 2.9. What is the difference of this number and its reciprocal?
Answer: 2.1

8. Toss-Up
The sum of six consecutive integers is -3. What is the largest of these integers?
Answer: 2

Bonus
An arithmetic sequence has ninth term $1\frac{1}{4}$ and fourteenth term $1\frac{2}{3}$. What is the first term of the sequence?
Answer: $\frac{7}{12}$

9. Toss-Up
If the expression $2^7 \bullet 4^6 \bullet 8^5$ is simplified to the form 2^N, what is the value of N?
Answer: 34

Bonus
What is the largest value of N such that 3^N is a factor of 20 factorial?
Answer: 8

10. Toss-Up
Simplify the ratio of 6 hours : 50 minutes.
Answer: 36 : 5

Bonus
A furlong is $\frac{1}{8}$ of a mile. An acre is $\frac{1}{640}$ of a square mile. How many acres are in a square furlong?
Answer: 10 [acres]

11. Toss-Up
What is the distance between the points (3, 8) and (11, 2)?
Answer: 10

Bonus
Circles centered at (2, 3) and (14, 8) are both tangent to the x-axis. What is the distance between the circles?
Answer: 2

12. Toss-Up
What is the area of a circle with radius π?
Answer: π^3

Bonus
The number of square inches in the surface area of a sphere is numerically equal to the number of cubic inches in the volume of the sphere. What is the radius of the sphere?
Answer: 3 inches

13. Toss-Up
Take the number of sides in a dodecagon, subtract the number of faces on a hexahedron, and then add the number of vertices in a rhombus. What is the result?
Answer: 10

Bonus
Let A and B be vertices of a cube so that the distance between A and B is as large as possible. A plane perpendicular to line segment AB passes through the midpoint of AB. What type of figure is formed by the intersection of this plane with the surface of the cube?
Answer: [regular] hexagon

14. Toss-Up
Solve for x: $4x + 18 = 6$.
Answer: [x =] -3

Bonus
What is the product of the solutions to the equation $x^2 - 5x - 14 = 0$?
Answer: -14

15. Toss-Up
In how many ways can the letters in the word OUNCE be rearranged?
Answer: 120

Bonus
In how many ways can the letters in the word MILLILITER be rearranged?
Answer: 100,800

MATH BOWL MATCH #5
Round 2 (November 4, 2024)

1. Toss-Up
Let X be the sum of the first ten even counting numbers. Let Y be the sum of the first ten odd counting numbers. What is X – Y?
Answer: 10

Bonus
Find the product of the quantity $1 - \frac{1}{2}$, the quantity $1 - \frac{1}{3}$, and the quantity $1 - \frac{1}{4}$.
Answer: $\frac{1}{4}$

2. Toss-Up
What is the area of a triangle with vertices at (3, 0); (0, 4); and (0, 0)?
Answer: 6

Bonus
What is the area of a quadrilateral with vertices at (1, 1); (6, 0); (4, 4); and (2, 7)?
Answer: 18.5

3. Toss-Up
The sum of two prime numbers is 80. What is the smallest possible value for the larger prime number?
Answer: 43

Bonus
Find the sum of the factors of 70.
Answer: 144

4. Toss-Up
Let X equal $2008^{2009} - 2009^{2008}$. Is X positive, negative, or neither?
Answer: positive

Bonus
Compute the difference between 2^{10} and 10^2.
Answer: 924

5. Toss-Up
A triangle has sides of length 5 units and 12 units. If the third side is an integer, what is the smallest possible perimeter of the triangle?
Answer: 25 [units]

Bonus
What is the length of the hypotenuse of a triangle with legs of length 20 inches and 21 inches?
Answer: 29 inches

6. Toss-Up
A rectangular candy wrapper measures 1 inch by 2 inches. How many candy wrappers can be cut from one square foot of paper?
Answer: 72

Bonus
Samantha works at a warehouse for a home-shopping television show. She is receiving a crate of jewelry cases, each measuring 6 inches by 2 inches by 1 inch. The dimensions of the crate are 4 feet by 3 feet by 2 feet. If the crate is fully packed, how many jewelry cases should be inside?
Answer: 3456 [jewelry cases]

7. Toss-Up

By what mixed number must you multiply $5\frac{1}{3}$ to obtain a product of 8?

Answer: $1\frac{1}{2}$

Bonus

A number is increased by a factor of $\frac{1}{2}$. The result is then increased by a factor of $\frac{1}{3}$. This results in a value of 84. What was the original number?
Answer: 42

8. Toss-Up
If m is an integer, and $m > 50$, what is the smallest possible integer value of $\sqrt{m}$?
Answer: 8

Bonus

If m is an integer, and $m > 50$, what is the smallest possible integer value of $\sqrt[3]{m^2}$?
Answer: 16

9. Toss-Up
Let S be the set of numbers with the following characteristics: the number must have two digits, the digits must be different, the sum of the digits must be divisible by 4, the number must not be prime, the number must not be divisible by 3. What is the largest number in set S?
Answer: 80

Bonus
Let T be the set of numbers with the following characteristics: the number must have three digits, the digits must all be different, no digit may be the factor of another digit, the number must be a perfect square. What is the largest number in set S?
Answer: 729

10. Toss-Up
What is twenty-five factorial divided by twenty-four factorial?
Answer: 25

Bonus
Solve for x: $(3!)(5!)(7!) = x!$
Answer: [x =] 10

11. Toss-Up
A row of marbles is laid out, starting with 5 red marbles, then 4 orange marbles, then 3 green marbles, then 2 blue marbles, then 1 black marble. The pattern then repeats with 5 red marbles, etc. What color is the 100th marble in the row?
Answer: green

Bonus
A bag contains 25 red marbles, 18 orange marbles, 12 green marbles, 9 blue marbles, and 6 black marbles. How many marbles must be drawn from the bag to guarantee drawing at least one marble of each color?
Answer: 65

12. Toss-Up
If $a * b$ means $5a + b^3$, what is $3 * 2$?
Answer: 23

Bonus
If $a * b$ means $5a + b^3$, solve the equation $x * 3 = 17$.
Answer: [x =] -2

13. Toss-Up
What is only two-digit number to be both a perfect square and a perfect cube?
Answer: 64

Bonus
The first five Fibonacci numbers are 1, 1, 2, 3, and 5. What is the twelfth Fibonacci number?
Answer: 144

14. Toss-Up
A cone and a cylinder share the same base, with the vertex of the cone located at the center of the other base of the cylinder. What fraction of the cylinder is inside the cone?
Answer: $\frac{1}{3}$

Bonus
The midpoints of rectangle VOTE are connected to form a quadrilateral. What specific type of quadrilateral is formed?
Answer: rhombus

15. Toss-Up
What is the largest prime factor of 81?
Answer: 3

Bonus
What is the sum of the prime factors of 23,023?
Answer: 54

MATH BOWL MATCH #5 2024

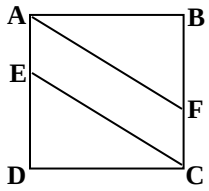
Problem Solving Round Option – Work as a team for MathCounts Practice!

1. Jimmy runs a mile in five minutes. Johnny runs a mile in seven minutes. Jughead runs a mile in nine minutes. If all three start a mile run at the same time, how far is Johnny ahead of Jughead when Jimmy finishes?

2. If twelve workers can lay 600 feet of pipe in five days, how long would it take for twenty workers to lay 9000 feet of pipe?

3. A figure is subjected to the following transformations in order: a translation of three units to the right, a dilation from the origin by a factor of four, a reflection about the x-axis, and a translation of two units down. If a point in the figure ends up at the origin, what were its original coordinates?

4. In the diagram at the right, square ABCD is intersected by parallel lines $\overleftrightarrow{AF}$ and $\overleftrightarrow{CE}$. If BF = 28 and CF = 17, find the distance between the parallel lines.



MATH BOWL MATCH #5 2024
Solutions to Problem Solving

1. Jimmy runs a mile in five minutes. Johnny runs a mile in seven minutes. Jughead runs a mile in nine minutes. If all three start a mile run at the same time, how far is Johnny ahead of Jughead when Jimmy finishes?

In six minutes, Jimmy is finished, Johnny has run $\frac{5}{7}$ of a mile, and Jughead has run $\frac{5}{9}$ of a mile. Simplifying $\frac{5}{7} - \frac{5}{9} =$ **$\frac{10}{63}$ of a mile**.

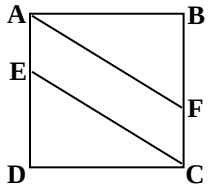
2. If twelve workers can lay 600 feet of pipe in five days, how long would it take for twenty workers to lay 9000 feet of pipe?

It takes 60 worker-days to lay 600 feet of pipe, so it will take fifteen times as many worker-days (900) to lay 6000 feet. Twenty workers would need to work 900 ÷ 20, or **45 days**.

3. A figure is subjected to the following transformations in order: a translation of three units to the right, a dilation from the origin by a factor of four, a reflection about the x-axis, and a translation of two units down. If a point in the figure ends up at the origin, what were its original coordinates?

Suppose the point starts at (x, y). Translating three units would lead to $(x + 3, y)$. Dilating by a factor of four leads to $(4x + 12, 4y)$. Reflecting about the x-axis leads to $(4x + 12, -4y)$. The final translation leads to $(4x + 12, -4y - 2)$. Setting these coordinates equal to $(0, 0)$ and solving, x is -3 and y is $\frac{-1}{2}$. So the original point is **$(-3, \frac{-1}{2})$**.

4. In the diagram at the right, square ABCD is intersected by parallel lines $\overleftrightarrow{AF}$ and $\overleftrightarrow{CE}$. If BF = 28 and CF = 17, find the distance between the parallel lines.



The side length of the square is 45, so its area is 2025 square units. The area of ΔABF is $\frac{1}{2}(28)(45) = 630$, as is the area of ΔCDE. So the area of parallelogram AFCE is 2025 – 1260 = 765. The Pythagorean Theorem shows that AF = 53 (28-45-53 triple), which is the base of the parallelogram if the height is the distance between the parallel lines. So this distance is **$\frac{765}{53}$ units**.