

## MATH BOWL MATCH #4, Round 1 (October 28, 2024)

### 1. Toss-Up

A rectangle has a perimeter of 20 feet. What is the largest possible area of the rectangle?

**Answer:** 25 square feet

### Bonus

One hundred twenty feet of fencing is to be used to surround two square gardens. The side length of the larger garden is twice that of the smaller garden. What is the area of the larger garden?

**Answer:** 400 square feet

### 2. Toss-Up

A spinner has twelve sections. Four are colored blue, three are colored yellow, two are colored green, and the rest are colored red. If the spinner is spun once, what is the probability that the result is red? Express your answer as a fraction in simplest form.

**Answer:**  $\frac{1}{4}$

### Bonus

A spinner has twelve sections. Four are colored blue, three are colored yellow, two are colored green, and the rest are colored red. If the spinner is spun twice, what is the probability that the results are blue and green, in either order? Express your answer as a fraction in simplest form.

**Answer:**  $\frac{1}{9}$

### 3. Toss-Up

What is the solution to the equation  $3X + 11 = 5$  ?

**Answer:**  $[X =] -2$

### Bonus

If  $2X + 3Y = 19$ , and  $3X + 4Y = 27$ , what is the value of  $X + Y$ ?

**Answer:** 8

### 4. Toss-Up

A triangle has side lengths that are whole numbers of centimeters. If two of the sides measure 3 centimeters and 7 centimeters, what is the longest possible length for the third side? **Answer:** 9 centimeters

### Bonus

An obtuse triangle has side lengths that are whole numbers of centimeters. If the two shorter sides measure 5 centimeters and 12 centimeters, what is the shortest possible length for the third side?

**Answer:** 14 centimeters

### 5. Toss-Up

NO CALCULATORS. Simplify the expression 3 plus 4 times 2 to the third power.

**Answer:** 35

**Bonus**

NO CALCULATORS. What is the largest two-digit number that is divisible by the sum of its digits?

**Answer:** 90

**6. Toss-Up**

A bicyclist travels at a rate of 30 miles per hour in the morning, then returns along the same route at 60 miles per hour in the afternoon. What is his average speed for his cycling for the day?

**Answer:** 40 miles per hour

**Bonus**

A train leaves New York heading for Chicago at 5:00am, traveling at 50 miles per hour. A second train leaves Chicago heading for New York at 7:00am, traveling at 60 miles per hour. The distance between the stations is 815 miles. At what time will the trains pass each other?

**Answer:** 1:30pm

**7. Toss-Up**

There is a 30% chance that it will rain today. What are the odds **against** it raining today?

**Answer:** 7 to 3

**Bonus**

The odds against it snowing on Monday are 10 to 1. The odds against it snowing on the next Monday are 8 to 1. What are the odds against it snowing on both of the two Mondays?

**Answer:** 98 to 1

**8. Toss-Up**

What is the value of  $3! + 2! + 1! + 0!$  ?

**Answer:** 10

**Bonus**

In how many ways can the letters in the word KENTUCKY be rearranged? **Answer:**  
20 160

**9. Toss-Up**

How many integers are less than 10 in absolute value?

**Answer:** 19

**Bonus**

What is the product of all integer solutions to  $|X + 5| < 2$  ?

**Answer:** -120

**10. Toss-Up**

NO CALCULATORS, NO PENCIL, NO PAPER. What is 50 divided by  $1/5$  ? **Answer:**  
250

**Bonus**

What is the largest proper factor of 2,011?

**Answer:** 1

**11. Toss-Up**

A rectangle has a diagonal of length 40 feet. If the length of the rectangle is 32 feet, find the width of the rectangle.

**Answer:** 24 feet

**Bonus**

A rectangle has a diagonal of length 26 feet. If its perimeter is 68 feet, find the area of the rectangle.

**Answer:** 240 square feet

**12. Toss-Up**

How many diagonals may be drawn in a convex decagon?

**Answer:** 35

**Bonus**

A triangle has vertices at the points (5, 8); (7, 14); and (12, 11). The three medians are drawn inside the triangle. What are the coordinates of the point where the medians intersect?

**Answer:** (8, 11)

**13. Toss-Up**

What is the slope of the line  $Y = 3X + 4$  ?

**Answer:** 3

**Bonus**

What is the abscissa of the X-intercept of the line  $3X + 7Y = 42$  ?

**Answer:** 14

**14. Toss-Up**

Three distinct whole numbers have a sum of 16. What is the largest possible product of these numbers? **Answer:** 140

**Bonus**

What is the sum of all the odd numbers between 1 and 1999, inclusive? **Answer:**  
1 000 000

**15. Toss-Up**

NO CALCULATORS. Find X to the Y power, if  $X = 64$  and  $Y = 1/2$ .

**Answer:** 8

**Bonus**

NO CALCULATORS. Find X to the Y power, if  $X = 27$  and  $Y = 2/3$ .

**Answer:** 9

## MATH BOWL MATCH #4, Round 2 (October 28, 2024)

### 1. Toss-Up

What is the tenth term in the sequence that begins 3, 7, 11, 15, and so on?

**Answer:** 39

### Bonus

Benjamin is given \$100 each month for an allowance. Franklin is given \$1 during the first month, \$2 during the second month, \$4 during the third month, with the amount continuing to double each month. After how many months will the total amount of all money Franklin has received be larger than that of Benjamin?

**Answer:** 10 [months] {Benjamin \$1000, Franklin \$1023}

### 2. Toss-Up

What is the greatest common factor of  $3XY^2$  and  $5X^2Y$ ?

**Answer:**  $XY$

### Bonus

Find all solutions to the equation  $X^2 - 8X + 12 = 0$ .

**Answer:** 2 [and] 6

### 3. Toss-Up

The digits 1, 2, 3, 4, 5, and 6 are used to make a six-digit number. What is the largest such number that is a multiple of 5?

**Answer:** 643 215

### Bonus

The digits 1, 2, 4, 7, 8, and 9 are used to make two three-digit numbers. What is the greatest possible difference between the two numbers?

**Answer:** 863 {987-124}

### 4. Toss-Up

The warehouse marks up prices by 40% over what the manufacturer charges. The retail store marks up prices by 30% over the warehouse price. Taxes of 10% are levied on the retail price. The result is the amount of money the customer must actually pay. What is the cost to the consumer of a hat for which the manufacturer charges \$10?

**Answer:** \$20.02

### Bonus

The warehouse marks up prices by 40% over what the manufacturer charges. The retail store marks up prices by 30% over the warehouse price. A non-profit organization receives a discount of  $X$  percent and pays no taxes. As a result, the non-profit pays one-tenth of one percent above what the manufacturer charges. What is the value of  $X$ ? **Answer:** 45

### 5. Toss-Up

What is the mean of the following numbers: 4, 5, 2, 6, 5, 1, 4, 5?

**Answer:** 4

**Bonus**

What is the mean deviation of the following numbers: 4, 5, 2, 6, 5, 1, 4, 5?

**Answer:** 1.25

**6. Toss-Up**

What is the term for the region inside a circle bounded by two radii and the circle itself?

**Answer:** sector

**Bonus**

What is the area of a sector of a circle formed by a 40-degree central angle if the diameter of the circle is 30 inches? Express your answer in terms of pi.

**Answer:** 100 pi square inches

**7. Toss-Up**

NO CALCULATORS. What is the largest three-digit power of two? **Answer:**

512

**Bonus**

What is the smallest four-digit number not containing the digit zero that is divisible by the sum of its digits?

**Answer:** 1116

**8. Toss-Up**

The remainder when a two-digit number is divided by six is one. The remainder when the number is divided by seven is two. What is the smallest possible value of the number?

**Answer:** 37

**Bonus**

The remainder when a two-digit number is divided by three is two. The remainder when the number is divided by five is two, and the remainder when the number is divided by seven is one. What is the number?

**Answer:** 92

**9. Toss-Up**

The two acute angles in a right triangle measure  $2X$  degrees and  $3X + 50$  degrees. What is the measure of the smallest angle in the triangle?

**Answer:** 16 degrees

**Bonus**

The angles of a triangle are in the ratio of 10 : 5 : 3. What is the measure of the largest angle?

**Answer:** 100 degrees

**10. Toss-Up**

Two water buffaloes will eat two tons of grass in two months. How long will it take six water buffaloes to eat three tons of grass?

**Answer:** one month

**Bonus**

One woodchuck would chuck four ricks of wood in ten days. How many woodchucks would it take to chuck twenty ricks of wood in two days?

**Answer:** 25 [woodchucks]

**11. Toss-Up**

Sweet potatoes cost 56 cents per pound. What is the cost of 8 ounces of sweet potatoes?

**Answer:** 28 cents

**Bonus**

An inchworm crawls 4 inches per minute. How many hours will the inchworm require to travel 10 yards?

**Answer:** 1.5 [hours]

**12. Toss-Up**

Solve for  $x$ :  $4x - 2 = 46$ .

**Answer:** [ $x =$ ] 12

**Bonus**

A pen costs twenty cents more than a pencil. If the cost of four pens and a pencil is two dollars, what is the cost of a pencil?

**Answer:** 24 cents

**13. Toss-Up**

What is the distance between the points (3, 8) and (8, -4)?

**Answer:** 13

**Bonus**

What is the length of the longest line segment that can be drawn inside a rectangular prism with dimensions 4 feet by 6 feet by 7 feet? Express your answer in simplest radical form.

**Answer:**  $\sqrt{101}$  feet

**14. Toss-Up**

Solve for  $X$ : 5 to the  $X$  power equals  $1/25$ .

**Answer:** [ $X =$ ] -2

**Bonus**

Solve for  $X$ : 2 to the power of  $3X - 1$  is equal to 1.

**Answer:** [ $X =$ ]  $1/3$

**15. Toss-Up**

Point A is located at (4, 1). Point B is the reflection of Point A over the X-axis. What are the coordinates of point B?

**Answer:** (4, -1)

**Bonus**

A triangle has vertices at  $(3, 4)$ ;  $(5, 9)$ ; and  $(10, 6)$ . What is the area of the triangle? **Answer:** 15.5 [square units]