

## MATH BOWL MATCH Week #3, Round 1 (October 21, 2024)

### 1. Toss-Up

The area of a square is 10 square meters. What is the length of its diagonal? Express your answer in simplest radical form.

**Answer:**  $2\sqrt{5}$  meters

### Bonus

A right triangle has two sides measuring 30 centimeters and 40 centimeters. What is the shortest possible length for the third side? Express your answer in simplest radical form.

**Answer:**  $10\sqrt{7}$  centimeters

### 2. Toss-Up

What is the only two-digit number that is both a perfect square and a perfect cube?

**Answer:** 64

### Bonus

Only two two-digit numbers have exactly three factors. What is the sum of these two numbers?

**Answer:** 74

### 3. Toss-Up

What is the median of the following set of numbers: 12, 12, 14, 14, 18, 10 ?

**Answer:** 13

### Bonus

Natalee has scored 81, 82, and 83 on his first three math tests. What average score must Natalee achieve on his next two tests to have a mean score of exactly 86?

**Answer:** 92

### 4. Toss-Up

Solve for  $x$ :  $2x + 11 = -9$ .

**Answer:** -10

### Bonus

Solve over the replacement set  $\{-2, -1, 0, 1, 2\}$ :  $x^2 > x$ .

**Answer:**  $\{-2, -1, 2\}$

### 5. Toss-Up

NO CALCULATORS. Simplify the expression 6 plus 7 times 8 minus 9.

**Answer:** 53

### Bonus

One row of Pascal's Triangle begins 1, 9, 36, etc. What is the largest number in this row?

**Answer:** 126

## 6. Toss-Up

What is the sum of the degree measures of the interior angles of a convex heptagon?

**Answer:** 900

### Bonus

What angle is formed by the hands of a clock at precisely 2:20?

**Answer:** 50 degrees

## 7. Toss-Up

There is a 30% chance that a raffle ticket wins a prize. Betty and David each have a raffle ticket. What is the probability that neither Betty nor David wins a prize?

**Answer:** 49%

### Bonus

There is a 30% chance that a raffle ticket wins a prize. Earl purchases five raffle tickets. To the nearest percent, what is the probability that Earl wins at least two prizes?

**Answer:** 47%

## 8. Toss-Up

What is the sum of the least common multiple of 4 and 6 and the greatest common factor of 4 and 6?

**Answer:** 14

### Bonus

One-fifth of the students spent one-fourth of the class period making graphs. If a total of 60 graphs were made, how many graphs could have been made if one-third of the students had made graphs for a one-half of the class period?

**Answer:** 200 [graphs]

## 9. Toss-Up

Pens cost 75 cents, and mechanical pencils cost 25 cents. If Will spends exactly \$5 on these two types of items, what is the smallest number of items that he could have purchased?

**Answer:** 8

### Bonus

Hamburgers cost \$2.50, and tuna salad sandwiches cost three dollars. If an order for twenty items cost 54 dollars, how many were hamburgers?

**Answer:** 12

## 10. Toss-Up

What is 20 divided by  $\frac{1}{2}$ ?

**Answer:** 40

### Bonus

The digits 2, 4, 5, and 7 are used to make two different two-digit numbers. If each digit is used exactly once, what is the greatest possible sum of the two two-digit numbers?

**Answer:** 126

**11. Toss-Up**

A right triangle has legs of lengths 10 centimeters and 24 centimeters. What is the perimeter of the triangle?

**Answer:** 60 centimeters

**Bonus**

A right triangle has two legs measuring 56 millimeters and 9 centimeters. How many meters long is the hypotenuse?

**Answer:** 0.106 [meters]

**12. Toss-Up**

A spinner is equally divided and numbered from 1 to 8. What is the probability of obtaining a composite number on two consecutive spins? Express your answer as a fraction in simplest form.

**Answer:** 9/64

**Bonus**

A spinner is equally divided and numbered from 1 to 8. What is the probability of obtaining three different numbers on three consecutive spins? Express your answer as a fraction in simplest form.

**Answer:** 21/32

**13. Toss-Up**

If  $f(x) = 4x + 6$ , for what value of  $x$  is  $f(x)$  negative ten?

**Answer:** -4

**Bonus**

If  $f(x) = 4x + 6$ , what is  $f(f(f(0)))$ ?

**Answer:** 126

**14. Toss-Up**

How many equilateral triangles make up the surface of an icosahedron?

**Answer:** 20

**Bonus**

A bicyclist takes 30 minutes to travel up a 5-mile hill and 10 minutes to travel back down. What the average speed for the round trip in miles per hour?

**Answer:** 15 [miles per hour]

**15. Toss-Up**

NO CALCULATORS. Find the square of the cube root of -64.

**Answer:** 16

**Bonus**

What is the largest solution to the equation  $x^2 - 3x = 70$  ?

**Answer:** 10

## MATH BOWL MATCH Week #3, Round 2 (October 21, 2024)

### 1. Toss-Up

NO CALCULATORS. What is the product of all integers less than 10 in absolute value?

**Answer:** 0

### Bonus

Consider the sequence 3, 5, 9, 17, 33, etc., where each successive term is one less than twice the preceding term. What is the twelfth term in this sequence?

**Answer:** 4097

### 2. Toss-Up

How many diagonals may be drawn in a convex hexagon?

**Answer:** 9

### Bonus

Nine points lie on a circle. Using these points as vertices, how many different quadrilaterals can be drawn?

**Answer:** 126

### 3. Toss-Up

Four numbers have a mean of 8, a median of 7, and a mode of 6. What is the largest of the four numbers?

**Answer:** 12

### Bonus

There are three urns containing marbles. The first urn contains 20% black marbles, with the rest white. The second urn contains 30% black marbles, with the rest white. The third urn contains 40% black marbles, with the rest white. If one marble is drawn randomly from each urn, what is the probability that at least one white and one black marble have been drawn? Express your answer as a percent.

**Answer:** 64%

### 4. Toss-Up

A dress is increased in price by 10%. A customer has a coupon for a 20% discount. If the dress originally cost \$200, how much did the customer pay?

**Answer:** \$176

### Bonus

A certain bank account yields 5% interest per year, compounded annually. Nate deposits \$1000 on January 2, 2000. At the start of each new year, he deposits an additional \$1000. To the nearest dollar, how much money is in the account at the end of five years?

**Answer:** \$5802

### 5. Toss-Up

What is the area of a regular quadrilateral with a perimeter of 8 centimeters?

**Answer:** 4 square centimeters

### Bonus

An isosceles right triangle has hypotenuse of length 20 inches. What is its area?

**Answer:** 100 square inches

### 6. Toss-Up

Find the smallest two-digit number that is not a factor of 24 factorial.

**Answer:** 29

### Bonus

One-half of Mrs. Smith's class chose an apple for a treat. One-fourth of the remaining students chose an orange, and one-third of the remaining students chose a grapefruit. The remaining 4 students did not receive a treat. How many students are in Mrs. Smith's class?

**Answer:** 16

### 7. Toss-Up

NO CALCULATORS! Compute four times three squared plus eight divided by two.

**Answer:** 40

### Bonus

Find the smallest whole number that cannot be written as the sum of three perfect squares of whole numbers.

**Answer:** 7

### 8. Toss-Up

The remainder when a two-digit number is divided by five is four. The remainder when the number is divided by four is one. What is the ones digit of the number?

**Answer:** 9

### Bonus

The remainder when a two-digit number is divided by three is one. The remainder when the number is divided by five is four, and the remainder when the number is divided by seven is three. What is the number?

**Answer:** 94

### 9. Toss-Up

In triangle ABC,  $AB = 5$ ,  $AC = 9$ , and  $BC = 8$ . List the angles (angle A, angle B, and angle C) in order of increasing measure.

**Answer:** angle C, angle A, angle B

### Bonus

Three concentric circles have radii in the ratio of 7 : 5 : 3. What is the ratio of the area of the annulus formed by the outermost two circles to the area of the annulus formed by the innermost two circles?

**Answer:** 3 : 2

### 10. Toss-Up

Quadrilateral ABCD is equiangular, with sides of integral length. If  $AB = 20$ , what is the smallest possible length of CD?

**Answer:** 20

### Bonus

The line  $y = 2x + 1$  does not pass through which quadrant?

**Answer:** [Quadrant] IV

**11. Toss-Up**

In how many ways can the letters in the word POTATO be rearranged?

**Answer:** 180

**Bonus**

A triangle has vertices at (3, 4); (4, 5); and (2, 1). What is the length of the longest side of the triangle? Express your answer in simplest radical form.

**Answer:**  $2\sqrt{5}$

**12. Toss-Up**

Solve for  $x$ :  $2x - 5 = 1$ .

**Answer:** [ $x =$ ] 3

**Bonus**

A pencil costs five cents more than an eraser. If the cost of two pencils and an eraser is forty cents, what is the cost of a pencil?

**Answer:** 15 cents

**13. Toss-Up**

Joey and Kyle are playing a game. A random number is chosen to start the game. Each turn, Joey adds three to the number to make a new number, then Kyle takes half of the resulting number. If the random number chosen at the start is 7, what will be the number after Kyle's second turn?

**Answer:** 4

**Bonus**

What is the fortieth term in the sequence that begins 4, 7, 10, 13, and so on?

**Answer:** 121

**14. Toss-Up**

If  $x$  and  $y$  are whole numbers, and  $x + 2y = 11$ , what is the largest possible value for  $y$ ?

**Answer:** [ $y =$ ] 5

**Bonus**

Find the reciprocal of the sum of the reciprocals of the four smallest counting numbers. Express your answer as a fraction in simplest form.

**Answer:**  $12/25$

**15. Toss-Up**

Point A is located at (3, 5). Point B is the reflection of Point A over the  $y$ -axis. What are the coordinates of point B?

**Answer:** (-3, 5)

**Bonus**

When a record store increases the price of its records by 40%, many people stop shopping there. To the nearest percent, by what percent must these prices be decreased in order to return to the original prices?

**Answer:** 29 [percent]