MATH BOWL MATCH Week #2
Round 1 (October 15, 2024)

1. Toss-Up
What is the solution to 3X plus 2 equals negative 10?
Answer: negative 4

Bonus
What is the sum of all whole numbers that are solutions to the inequality $7X + 12 < 47$?
Answer: 10

2. Toss-Up
The second term of an geometric sequence is 15. The fifth term is 3. What is the eighth term of the sequence?
Answer: 3/5 or 0.6

Bonus
There are ten numbers that form an arithmetic sequence. The second number is 40, and the sixth number is 100. What is the sum of all ten numbers?
Answer: 925

3. Toss-Up
Let S be the set of all numbers with the following properties. To be in S, a number must be rational, positive, and less than 50. Finally, when simplified as a fraction, the number must have a denominator of 5. How many numbers are in S?
Answer: 200

Bonus
Let T be the set of all four-digit numbers with the following properties. To be in T, a number must be a perfect square and be odd. How many numbers are in T?
Answer: 34

4. Toss-Up
Find the measure of each interior angle of a regular decagon.
Answer: 144 degrees

Bonus
An altitude is drawn from the right angle of a right triangle with sides of lengths 15 centimeters, 20 centimeters, and 25 centimeters. What is the length of this altitude?
Answer: 24 centimeters

5. Toss-Up
What is the final cost of a $45 pair of jeans after applying a 20% discount and 6% sales tax?
Answer: $38.16

Bonus
After adding an 18% tip, Jeremy's total bill was $14.75. What was the cost before adding the tip?
Answer: $12.50

6. Toss-Up
NO CALCULATORS. Find the sum of the following numbers: zero to the power of one, one to power of two, two to the power of three, three to the power of four, four to the power of zero.
Answer: 91

Bonus
NO CALCULATORS. Compute 624 times 624 minus 619 times 629.
Answer: 25

7. Toss-Up
What is the number of faces on an icosahedron?
Answer: 20

Bonus
A doubly stellated icosahedron has 180 faces and 92 vertices. How many edges does this solid have?
Answer: 270

8. Toss-Up
If $f(x) = 2x^3 + 1$, what is the value of $f(4)$?
Answer: 129

Bonus
What is the smallest whole number x for which $0.2x^2 + 25x - 2011$ is positive?
Answer: 56

9. Toss-Up
What is the smallest natural number with exactly five factors?
Answer: 16

Bonus
What is the sum of all prime numbers between 90 and 110?
Answer: 517

10. Toss-Up
NO CALCULATORS. NO PENCIL. NO PAPER. What is the product of eight, nine, ten, and eleven?
Answer: 7920

Bonus
NO CALCULATORS. NO PENCIL. NO PAPER. What is the largest five-digit number consisting of all odd digits that is not a multiple of three?
Answer: 99 997

11. Toss-Up
How far is the point (8, 2) from the point (3, 14)?
Answer: 13 [units]

Bonus
One endpoint of a line segment lies at (-3, -4). The midpoint of the line segment is at (-1, 2). What are the coordinates of the other endpoint of the line segment?
Answer: (1, 8)

12. Toss-Up
The remainder when a two-digit number is divided by nine is two. What is the largest possible value for the number?
Answer: 92

Bonus
The remainder when a two-digit number is divided by three is two. The remainder when the number is divided by five is three, and the remainder when the number is divided by seven is four. What is the number?
Answer: 53

13. Toss-Up
The angles of a triangle measure X degrees, X+10 degrees, and 2X+30 degrees. What is the measure of the largest angle?
Answer: 100 degrees

Bonus
A pentagon contains two right angles and one eighty-degree angle. The remaining two angles are congruent obtuse angles. What is the measure of each of the congruent obtuse angles?
Answer: 140 degrees

14. Toss-Up
Four data values have a mean of 5, a median of 6, and a mode of 7. If none of the data values is negative, what is the largest possible value for the largest data value?
Answer: 7

Bonus
What is the median value of the first 25 prime numbers?
Answer: 41

15. Toss-Up
What is 12 divided by 2/3?
Answer: 18

Bonus
A detective is checking the license plate of a car. The first three characters are digits (0 through 9), and he knows the first digit is 4, the second digit is even, and no two digits are the same. The last three characters are letters, and he knows the first is one of the five vowels, the second is not a vowel, and the third is Q. How many different license plates satisfy these conditions?
Answer: 3360

MATH BOWL MATCH #2
Round 2 (October 15, 2024)

1. Toss-Up
If the longest possible chord in a circle measures 14 feet, what is the area of the circle?
Answer: 49π square feet

Bonus
The diameter of a sphere is increased by 20 percent. By what percent is the surface area of the sphere increased?
Answer: 44 [percent]

2. Toss-Up
The measure of one acute angle of a right triangle is 30 degrees less than half the measure of the other acute angle. What is the degree measure of the smallest angle in the triangle?
Answer: 10 [degrees]

Bonus
What angle is made by the hands of a clock at 6:30?
Answer: 15 degrees

3. Toss-Up
The natural number X is a composite number. X+1 is a perfect square, and X+2 is a prime number. What is the smallest possible value for X?
Answer: 15

Bonus
The length in centimeters of each side of a scalene triangle is a perfect square. What is the smallest possible value for the perimeter of the triangle?
Answer: 77 centimeters

4. Toss-Up
A frog is trying to jump out of a greasy pipe that is twenty feet long. Each time the frog jumps, he moves three feet. However, before he can jump again, he slips backward two feet. How many times must the frog jump to escape the pipe?
Answer: 18

Bonus
An abundant number is a positive integer such that the sum of its proper divisors is greater than the number itself. For example, 12 is abundant because 1+2+3+4+6 is greater than 12. What is the smallest abundant number that is not a multiple of 6?
Answer: 20

5. Toss-Up
If A equals two times B plus five and B equals two times C plus four, express A in terms of C.
Answer: [A equals] 4C + 13

Bonus
An RC car is discounted in price by 25%. Sam has a coupon that discounts the sale price by an additional 25%. What percent of the original price must Sam pay for the RC car?
Answer: 56.25 [percent]

6. Toss-Up
Find the area of a rectangle measuring two feet by three inches.
Answer: 1/2 square foot OR 72 square inches

Bonus
ABCDEFGH is a regular octagon. What is the measure of angle CDF?
Answer: 112.5 degrees

7. Toss-Up
The operation A * B [read "A star B"] is to be defined as A times B minus 10. What is the difference between the values of 6 * 7 and 7 * 6?
Answer: 0

Bonus
The operation A * B [read "A star B"] is to be defined as A times B minus 10. What is the smallest solution to the equation X * X equals X plus 2?
Answer: negative 3 {4 is the other solution}

8. Toss-Up
The arithmetic mean of five distinct whole numbers is 11. What is the smallest possible value of the largest number?
Answer: 13

Bonus
What is the smallest number that is both ten more than a prime number and ten less than a prime number, without itself being prime?
Answer: 21

9. Toss-Up
The area of a rectangle is 60 square meters. If the length is 12 meters, what is its perimeter?
Answer: 34 meters

Bonus
The area of a rectangle is 120 square feet. If the diagonal is 17 feet, what is its perimeter?
Answer: 46 feet

10. Toss-Up
A spinner is equally divided and numbered from 1 to 20. What is the probability of spinning a composite number on the spinner? Express your answer as a fraction in simplest form.
Answer: 11/20

Bonus
Two standard number cubes are rolled simultaneously. What is the probability that the product of the two numbers is 12? Express your answer as a fraction in simplest form.
Answer: 1/9

11. Toss-Up
If $x = 3$, then $f(x) = 30$. If $x = 6$, then $f(x) = 600$. Assuming the function is linear, what is $f(x)$ if $x = 5$?
Answer: 410

Bonus
What is the thirtieth term in the arithmetic sequence that begins 1, 5, 9, 13, and so on?
Answer: 117

12. Toss-Up
NO CALCULATORS. Find the product of 612,345 and 11.
Answer: 6,735,795

Bonus
What is the sum of the prime factors of 999?
Answer: 40

13. Toss-Up
What is the smallest integral solution to $9x > -6$?
Answer: 0

Bonus
What is the product of all integral solutions to the equation $x^4 = 81$?
Answer: -9

14. Toss-Up
What is the probability that a randomly chosen two-digit number is less than 40? Express your answer as a fraction in simplest form.
Answer: 1/3

Bonus
A drawer contains six white socks and six black socks. Two socks are drawn at random. What is the probability that the socks are the same color? Express your answer as a fraction in simplest form.
Answer: 5/11

15. Toss-Up
In which quadrant does the point (-1, 4) lie?
Answer: (Quadrant) II [OR second quadrant]

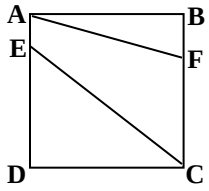
Bonus
The graph of the function $f(x) = 2x - 1$ passes through which quadrants?
Answer: (Quadrants) I, III, and IV

MATH BOWL MATCH #2,
Problem Solving (October 15, 2024)

1. Three whole numbers have a product of 2012. What is the smallest possible difference between the largest and smallest number?

2. Two thousand twelve points are equally spaced on a circle. Three points are chosen at random. What is the probability that when the points are connected the result will be a right triangle?

3. In the diagram at the right, ABCD is a square. If E is the trisection point of AD closest to A, and F is the trisection point of BC closest to B, what fraction of the area of the square lies inside quadrilateral AFCE?



MATH BOWL MATCH #2
Problem Solving Answers (October 15, 2024)

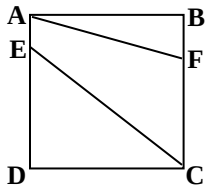
1. Three whole numbers have a product of 2012. What is the smallest possible difference between the largest and smallest number?

   The prime factorization of 2012 is $2 \cdot 2 \cdot 503$, and this is the closest the three factors can be. The resulting difference is $503 - 2 = \mathbf{501}$.

2. Two thousand twelve points are equally spaced on a circle. Three points are chosen at random. What is the probability that when the points are connected the result will be a right triangle?

   A right triangle will result if and only if two of the points lie on a diameter. There are 1006 possible diameters, with 2010 other points available for each diameter. This gives $1006 \cdot 2010 = 2022060$ ways to select a right triangle, out of ${}_{2012}C_3 = \dfrac{2012 \cdot 2011 \cdot 2010}{1 \cdot 2 \cdot 3} = 1\,355\,454\,220$. Simplifying this ratio, $\dfrac{2022060}{1355454220} = \dfrac{\mathbf{110103}}{\mathbf{67772711}}$.

3. In the diagram at the right, ABCD is a square. If E is the trisection point of AD closest to A, and F is the trisection point of BC closest to B, what fraction of the area of the square lies inside quadrilateral AFCE?



   Let the side length of the square be s. The area of $\Delta ABF = \frac{1}{2}bh = \frac{1}{2} \cdot AB \cdot BF = \frac{1}{2} \cdot s \cdot \frac{1}{3}s = \frac{1}{6}s^2$. The area of $\Delta EDC = \frac{1}{2}bh = = \frac{1}{2} \cdot CD \cdot DE = \frac{1}{2} \cdot s \cdot \frac{2}{3}s = \frac{1}{3}s^2$. Adding these expressions, the sum of the triangle areas is $\frac{1}{2}s^2$, which is $\frac{1}{2}$ of the area of the square. Thus, the remaining $\mathbf{\frac{1}{2}}$ of the square lies inside quadrilateral AFCE.