

## MATH BOWL MATCH #1, Round 1 (Week of October 7, 2024)

### 1. Toss-Up

The first term of an arithmetic sequence is 20. The fourth term is 8. What is the eighth term of the sequence? **Answer:** -8

### Bonus

In a geometric sequence, the ratio of the first term to the third term is 64 to 1. What is the ratio of the sixth term to the third term? **Answer:** 512 to 1

### 2. Toss-Up

Tommy is collecting numbers. To be in his collection, a number must be positive and less than 100. The number must also be rational. Finally, when simplified as a fraction, the number must have a denominator of 2. How many numbers are in Tommy's collection? **Answer:** 100

### Bonus

How many rational numbers between 0 and 1 will have a denominator of 100 when written in simplest fraction form? **Answer:** 40

### 3. Toss-Up

What is the final cost of a \$80 pair of tennis shoes after applying a 10% discount and 6% sales tax?

**Answer:** \$76.32

### Bonus

Gina paid \$8.05 for her dinner and left a 20% tip. The cost of Hank's meal, including tip, was the same as that of Gina, including tip, but Hank only left a 15% tip. What was the cost of Hank's dinner before the tip?

**Answer:** \$8.40

### 4. Toss-Up

What is the area of a regular quadrilateral with a perimeter of 4 centimeters? **Answer:** 1 square centimeter

### Bonus

An isosceles right triangle has hypotenuse of length 18 inches. What is its area? **Answer:** 81 square inches

### 5. Toss-Up

NO CALCULATORS. Find the sum of the following numbers: one to power of nine, nine to the power of one, zero to the power of nine, nine to the power of zero. **Answer:** 11

### Bonus

NO CALCULATORS. Compute 475 times 475 minus 472 times 478.

**Answer:** 9

### 6. Toss-Up

Which of the five Platonic solids has the smallest number of faces?

**Answer:** tetrahedron

**Bonus**

A triply stellated icosahedron has 540 faces and 272 vertices. How many edges does this solid have?

**Answer:** 810

**7. Toss-Up**

In a certain lottery, each participant selects two different whole numbers between 1 and 100, inclusive. To win the jackpot, these two numbers must match two randomly-selected numbers. The probability of winning the jackpot is one out of  $N$ . What is the value of  $N$ ?

**Answer:** 4950

**Bonus**

In a certain lottery, each participant selects three different whole numbers between 1 and 100, inclusive. To win a prize, exactly two of the three numbers must match two of three randomly-selected numbers. The probability of winning a prize is one out of  $N$ . What is the value of  $N$  to the nearest whole number?

**Answer:** 556

**8. Toss-Up**

If  $f(x) = 3x^2 + 1$ , what is the value of  $f(10)$ ?

**Answer:** 301

**Bonus**

What is the smallest whole number  $x$  for which  $0.1x^2 + 10x - 2011$  is positive? **Answer:** 101

**9. Toss-Up**

What is the smallest natural number with exactly six factors?

**Answer:** 12

**Bonus**

What is the sum of all prime numbers between 110 and 130?

**Answer:** 240

**10. Toss-Up**

NO CALCULATORS. NO PENCIL. NO PAPER. What is the product of the eight smallest whole numbers?

**Answer:** 0

**Bonus**

NO CALCULATORS. NO PENCIL. NO PAPER. What is the product of the eight smallest natural numbers?

**Answer:** 40320

### 11. Toss-Up

If I get an A in math class, then my uncle will buy me a new car. If I study well for the test, then I will get an A in math class. What conclusion can be drawn from these two statements?

**Answer:** If I study well for the test, then my uncle will buy me a new car.

### Bonus

What is the term for the locus of points equidistant from a given point in space? **Answer:** sphere

### 12. Toss-Up

How far is the point (2, 4) from the point (6, 1)?

**Answer:** 5 [units]

### Bonus

One endpoint of a line segment lies at (3, 4). The midpoint of the line segment is at (2, 10). What are the coordinates of the other endpoint of the line segment? **Answer:** (1, 16)

### 13. Toss-Up

A triangle has side lengths of 5 centimeters and 12 centimeters. What is the longest possible length of the third side, if it is an integral number of centimeters?

**Answer:** 16 centimeters

### Bonus

The angles of a triangle measure  $5X+20$  degrees,  $6X+13$  degrees, and  $7X+3$  degrees. What is the measure of the largest angle?

**Answer:** 61 degrees

### 14. Toss-Up

NO CALCULATOR. Compute 60 minus 36 divided by 6 times 3 plus 1.

**Answer:** 43

### Bonus

Consider the problem 20 plus 10 divided by 5 times 2 minus 7. Abby does the operations from left to right to obtain her answer. Bob does multiplication, then division, then addition, then subtraction to obtain his answer. Carla correctly does order of operations to get her answer.

What is the difference between the largest and smallest of the three answers?

**Answer:** 12

### 15. Toss-Up

Eight aardvarks can eat eight hundred ants in eight minutes. How many aardvarks would it take to eat two thousand four hundred ants in four minutes?

**Answer:** 48

**Bonus**

To make orange juice, Sasha mixes 16 ounces of juice concentrate with 40 ounces of water. This makes 56 ounces of juice. Sasha wants to make 84 ounces of juice. How many ounces of juice concentrate will she need?

**Answer:** 24 [ounces]

## MATH BOWL MATCH #1, Round 2 (October 7, 2024)

### 1. Toss-Up

A spinner is equally divided and numbered from 1 to 12. What is the probability of spinning a prime number on the spinner? Express your answer as a fraction in simplest form. **Answer:**  $\frac{5}{12}$

### Bonus

The letters in the word "PIZZA" are written on separate cards and placed in a bag. Two cards are drawn simultaneously at random. What is the probability that exactly one of the drawn cards shows an Z? Express your answer as a fraction in simplest form. **Answer:**  $\frac{3}{5}$

### 2. Toss-Up

What is the smallest four-digit perfect square?

**Answer:** 1024 **Bonus**

Simplify:  $\sqrt{3} \square \sqrt{6} \square \sqrt{8}$

**Answer:** 12

### 3. Toss-Up

Four data values have a mean of 4, a median of 4, and a mode of 4. If none of the data values is negative, what is the largest possible value for the largest data value? **Answer:** 8

### Bonus

Beth has taken twelve tests. The mean of her seven lowest test grades is 84. The mean of her seven highest test grades is 96. The median of her test grades is 93. What is the mean of her test grades? Express your answer as a decimal to the nearest tenth. **Answer:** 89.5

### 4. Toss-Up

A cube of volume 64 cubic inches is painted, then cut into one-inch cubes. What fraction of the one-inch cubes have paint on exactly one sides? Express your answer in simplest form. **Answer:**  $\frac{3}{8}$

### Bonus

A cube of volume 64 cubic inches is painted, then cut into one-inch cubes. A randomly chosen one-inch cube is rolled. What is the probability that the side facing up is painted? **Answer:**  $\frac{1}{4}$

### 5. Toss-Up

What is 20 divided by  $\frac{1}{5}$ ?

**Answer:** 100

### Bonus

Carrie has a lock with a four-digit combination key (each digit can be 0 through 9). She remembers that there are no adjacent digits that are the same, and the last number is even. How many such combinations are possible? **Answer:** 3645

### 6. Toss-Up

What is the length of the longest chord in a circle of radius 4 feet? **Answer:**  
8 feet

### Bonus

A chord of length 8 centimeters lies in a circle of radius 5 centimeters. What is the distance from the chord to the center of the circle?

**Answer:** 3 centimeters

### 7. Toss-Up

A rooster that is 20 inches tall casts a 55-inch shadow. At the same time, a dog casts a shadow that is 5 and one-half feet long. How tall is the dog?

**Answer:** 2 feet

### Bonus

The area of a circle is 40 square meters. A second circle has a diameter that is 50% larger than the diameter of the first circle. What is the area of the second circle? **Answer:** 90 square meters

### 8. Toss-Up

How many diagonals lie inside a regular hexagon? **Answer:**  
9

### Bonus

What is the sum of the measures of the interior angles of a hexagon? **Answer:**  
720 degrees

### 9. Toss-Up

What is the units digit of  $2011^{2011}$ ?

**Answer:** 1

### Bonus

What is the hundreds digit of  $2011^{2011}$ ?

**Answer:** 1

### 10. Toss-Up

The measure of one acute angle of a right triangle is 6 degrees more than half the measure of the other acute angle. What is the degree measure of the smallest angle in the triangle?

**Answer:** 34 [degrees]

### Bonus

The four angles of a quadrilateral have the following degree measures:  $2x + 4$ ,  $3x + 5$ ,  $4x + 6$ , and  $5x + 9$ . What is the degree measure of the largest angle in the quadrilateral? **Answer:** 129 [degrees]

**11. Toss-Up**

Jason drove at an average speed of 56 miles per hour. He left at 7:25am and arrived at 9:10am. How many miles did Jason drive?

**Answer:** 98 [miles]

**Bonus**

Kellie drove for 20 minutes at an average speed of 54 miles per hour. She then drove for 40 minutes at an average speed of 66 miles per hour. What was his average speed during these 60 minutes?

**Answer:** 62 miles per hour

**12. Toss-Up**

What angle is made by the hands of a clock at 9:00?

**Answer:** 90 degrees

**Bonus**

What angle is made by the hands of a clock at 4:20?

**Answer:** 10 degrees

**13. Toss-Up**

A bag contains 5 blue marbles, 4 red marbles, 3 yellow marbles, 2 green marbles, and 1 white marble. How many marbles must be drawn to guarantee having at least one of each color?

**Answer:** 15

**Bonus**

A bag contains 5 blue marbles, 4 red marbles, 3 yellow marbles, 2 green marbles, and 1 white marbles. You draw one marble at random with your left hand. While holding that marble, you draw a second marble with your right hand. What is the probability that the marble in your left hand is the same color as the marble in your right hand? Express your answer as a fraction in simplest form.

**Answer:**  $\frac{4}{21}$

**14. Toss-Up**

NO CALCULATORS, NO PENCILS, NO PAPER. What is the smallest four-digit number that contains no odd digits and in which no digit is used twice? **Answer:** 2046

**Bonus**

What is the largest odd factor of 320?

**Answer:** 5

**15. Toss-Up**

There are 640 acres in a square mile. There are 8 furlongs in a mile. How many acres are in a square furlong?

**Answer:** 10

**Bonus**

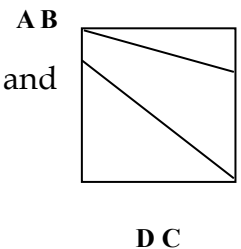
How many two-digit prime numbers are there?

**Answer:** 21

**MATH BOWL MATCH #1, Problem Solving (October 7, 2024)**

1. How many zeros are at the end of 2011! ?
2. Two integers have a product of 2011. What is the smallest possible difference between the numbers?
3. Two thousand eleven points lie on a circle. How many triangles can be formed using these points as vertices?

4. In the diagram at the right, ABCD is a square. If  $AE = \frac{1}{2011}AD$  and  $BF = \frac{1018}{2011}BC$ , what fraction of the area of the square lies inside quadrilateral AFCE?



5. A jeweler has a pouch containing gemstones. He would have to remove 23 stones without looking to guarantee that he produced a diamond. He would have to remove 24 stones without looking to guarantee that he produced a ruby. He would have to remove 26 stones without looking to guarantee that he produced a sapphire. He would have to remove 33 stones without looking to guarantee that he produced a jacinth. How many gemstones are in the bag?

**MATH BOWL MATCH #1, Problem Solving Answers (October 7, 2024)**

1. How many zeros are at the end of 2011! ?

A zero occurs for each factor of 10, which would be the smaller of the number of factors of 2 or 5 (there are fewer factors of 5).  $2011! = 2011 \times 2010 \times 2009 \times \dots \times 3 \times 2 \times 1$ , and there are 402 of these factors that are multiples of 5. Of these 80 have a second factor of 5 (like 25, 50, etc.), 16 have a third, and 3 have a fourth. This is a total of 501 factors of 5 (the prime factorization of 2011! has an exponent of 501 on the 5). Thus there are **501** zeros at the end.

2. Two integers have a product of 2011. What is the smallest possible difference between the numbers?

Since 2011 is prime, the factors of 2011 are 1 and 2011. The only possible difference is  $2011 - 1 = \mathbf{2010}$ .

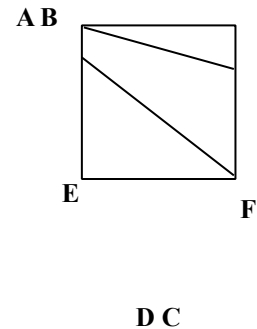
3. Two thousand eleven points lie on a circle. How many triangles can be formed using these points as vertices?

Any three of these points will form a triangle, since no three points on a circle lie on the same line. The number of ways to select three points from a set of 2011 is

$$C = \frac{2011 \times 2010 \times 2009}{1 \times 2 \times 3}$$

= **1 353 633 165**.

4. In the diagram at the right, ABCD is a square. If  $AE = \frac{1}{2011}AD$  and



$BF = \frac{1018}{2011}BC$ , what fraction of the area of the square lies inside quadrilateral AFCE?

Let the side length of the square be  $s$ . The area of  $\triangle ABF = \frac{1}{2}bh = \frac{1}{2} \times AB \times BF = \frac{1}{2} \times s \times \frac{1018}{2011}s = \frac{1018}{2011}s^2$ .

The area of  $\triangle EDC = \frac{1}{2}bh = \frac{1}{2} \times CD \times DE = \frac{1}{2} \times s \times \frac{1}{2011}s = \frac{1}{2011}s^2$ . Adding these expressions, the sum of the triangle areas is  $\frac{1019}{2011}s^2$ , which is  $\frac{1019}{2011}$  of the area of the square. Thus, the remaining  $\frac{1002}{2011}$  of the square lies inside quadrilateral AFCE.

5. A jeweler has a pouch containing gemstones. He would have to remove 23 stones without looking to guarantee that he produced a diamond. He would have to remove 24 stones without looking to guarantee that he produced a ruby. He would have to remove 26 stones without looking to guarantee that he produced a sapphire. He would have to remove 33 stones without looking to guarantee that he produced a jacinth. How many gemstones are in the bag?

Let  $D$  be the number of diamonds,  $R$  the number of rubies,  $S$  the number of sapphires, and  $J$  the number of jacinths. Since 23 stones must be removed to guarantee a diamond, 22 of the stones must not be diamonds. Thus,  $R + S + J = 22$ . Similar arguments results in the following equations:  $D + S + J = 23$ ,  $D + R + J = 25$ , and  $D + R + S = 32$ . Adding all four equations results in  $3D + 3R + 3S + 3J = 102$ . Dividing both sides by 3,  $D + R + S + J = 34$ . But this is all the gemstones, so there are **34** gemstones in the bag.