Activity

10-1 Operations With Matrices

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MODEL & DISCUSS

This screen shows the number of Small, Medium, Large, and Extra Large limited-edition silkscreen shirts on sale at an online boutique.

A. Construct a table to summarize the inventory that is on sale.

B. At the end of the day, the boutique has sold this many of each T-shirt from the sale items: red: 4 S, 6 M, 3 L, 5 XL; blue: 2 S, 8 M, 4 L, 0 XL. Make two new tables, one showing the merchandise sold and one showing the inventory that is left.

C. **Look for Relationships** What relationships did you use in creating the two tables in Part B?

HABITS OF MIND

Make Sense and Persevere Edwin summarized the information given in the problem in 2 rows and 4 columns. In Part B, he summarized the information about the number of shirts sold in a table that had 4 rows and 2 columns. Was this organizational strategy helpful? Explain.

EXAMPLE 1 **Try It! Represent Data With a Matrix**

1. In matrix C, the entries are the numbers of students on a committee. Column 1 lists girls, column 2 lists boys, row 1 lists sophomores, and row 2 lists juniors. Find a_{12}, a_{21}, and a_{22}, and tell what each number represents.

$$C = \begin{bmatrix} 7 & 5 \\ 8 & 10 \end{bmatrix}$$

EXAMPLE 2 **Try It! Apply Scalar Multiplication**

2. In this matrix C, the rows represent prices for shirts and khakis. The columns have the same meaning as in Example 2. If the sales tax rate is 6%, use scalar multiplication to find the sales tax for each item.

$$C = \begin{bmatrix} 75 & 40 & 25 \\ 100 & 60 & 30 \end{bmatrix}$$

HABITS OF MIND

Generalize Let the dimensions of matrix Z be 3×4. After multiplying this matrix by a scalar, what are the dimensions of the product matrix? Explain.

Notes

Assess

EXAMPLE 3 **Try It! Add and Subtract Matrices**

3. Consider matrices M and N.

$$M = \begin{bmatrix} -3 & 5 \\ 2 & 0 \end{bmatrix}, N = \begin{bmatrix} 6 & 5 \\ -8 & 0.2 \end{bmatrix}$$

a. What are matrices $M + N$ and $N + M$?

b. What are matrices $M - N$ and $N - M$?

EXAMPLE 4 **Try It! Understand Matrix Addition and Subtraction**

4. Consider the matrices below.

$$P = \begin{bmatrix} 5 & 2 & -3 \\ 7 & 0 & -5 \end{bmatrix}, Q = \begin{bmatrix} 2 & -2 \\ 5 & -5 \\ -7 & 7 \end{bmatrix}, R = \begin{bmatrix} 6 & 0.5 \\ -3 & 0 \\ -2 & -2 \end{bmatrix}$$

a. Find $R - Q$. What other matrix sums or differences can be calculated?

b. Find the additive inverses of P, Q, and R.

HABITS OF MIND

Communicate Precisely What must be true about two matrices for their sum or difference to exist?

EXAMPLE 5 **Try It! Use Matrices to Translate and Dilate Figures**

5. A segment has endpoints $M(8, -7)$ and $N(1, 2)$.

a. Use matrices to represent a translation of $\overline{MN}$ to $\overline{RS}$ by 6 units left and 3 units down. What are the coordinates of R and S?

b. Use matrices to represent a dilation of $\overline{MN}$ to $\overline{DE}$ by a scale factor of 3, centered at the origin. What are the coordinates of D and E?

HABITS OF MIND

Model With Mathematics The matrix $T = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 2 \end{bmatrix}$ represents a triangle. Use matrices to determine whether dilating by a factor of 2 and then translating 5 units right is the same as translating the triangle 5 units right and then dilating by a factor of 2. Does the order of the transformations matter? Explain.

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you interpret matrices and operate with matrices?

2. **Error Analysis** Tonya says $\begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ would produce a zero matrix. Explain her error.

3. **Communicate Precisely** Explain how you know if two matrices can be added. Then explain how to add them.

4. **Vocabulary** What are equal matrices? Give an example of equal matrices.

Do You KNOW HOW?

Identify the element for each matrix.

5. $\begin{bmatrix} 4 & 1 & 0 \\ 7 & 3 & 5 \end{bmatrix}$; a_{23}

6. $\begin{bmatrix} -6 \\ 2 \end{bmatrix}$; a_{11}

Given $A = \begin{bmatrix} 3 & -2 \\ 7 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 7 \\ -4 & 12 \end{bmatrix}$, calculate each of the following.

7. $A + B$

8. $B - A$

9. $4A$

10. $A - B$

11. The endpoints of $\overline{AB}$ are represented by the matrix $\begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix}$.
Find the image of the segment after a dilation, centered at the origin, by a scale factor of 2.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

12. **Communicate Precisely** Explain how you would solve for each variable. Then find the value of each variable. $\begin{bmatrix} a & b-3 \\ c & d+5 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 6 & 4 \end{bmatrix}$

13. **Make Sense and Persevere** Find the sum of $A = \begin{bmatrix} 5 \\ 3 \\ 8 \end{bmatrix}$ and the additive inverse of $P = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$

14. **Error Analysis** Describe and correct the error a student made in translating the points $A(1, -3)$, $B(2, 1)$ and $C(-3, -2)$ 3 units left and 1 unit up.

Original points $\begin{pmatrix} 1 & 2 & -3 \\ -3 & 1 & -2 \end{pmatrix}$

Translation matrix $\begin{pmatrix} -3 & -3 & -3 \\ -1 & -1 & -1 \end{pmatrix}$

Answer matrix $\begin{pmatrix} -2 & -1 & -6 \\ -4 & 0 & -3 \end{pmatrix}$ ✗

15. **Construct Arguments** Suppose A and B are two matrices with the same dimensions. Explain how to find $A + B$, $A - B$, and matrix C such that $A + C$ is the zero matrix.

16. **Higher Order Thinking** Explain why $A = \begin{bmatrix} 0.5 \\ 4 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} \\ 1+3 \end{bmatrix}$ have the same additive inverse.

17. **Mathematical Connections** For the set of real numbers, if the sum of two numbers is the additive identity element, then the two numbers are additive inverses of each other. How does this property relate to matrix addition?

18. **Reason** The coordinates of the vertices of a square are represented in a matrix. The matrix is then multiplied by the scalar 3. How does the area of the new square compare to the area of the original square?

PRACTICE & PROBLEM SOLVING

PRACTICE

19. In matrix D, the entries are the number of students playing volleyball at a high school. Column 1 lists boys, column 2 lists girls, row 1 lists juniors, and row 2 lists seniors. Find d_{22}, d_{12}, and d_{11}, and tell what each number represents. $D = \begin{bmatrix} 4 & 5 \\ 7 & 6 \end{bmatrix}$ SEE EXAMPLE 1

20. In the price matrix P, the rows represent prices for sweatshirts and sweatpants. The columns represent the color scheme of the items: white, red, and tie-dye. If the sales tax rate is 7%, find the sales tax of each item.

$P = \begin{bmatrix} 30 & 40 & 50 \\ 25 & 35 & 55 \end{bmatrix}$ SEE EXAMPLE 2

Given matrices $X = \begin{bmatrix} 7 & 2 & 1 \\ 4 & -3 & 6 \end{bmatrix}$, $Y = \begin{bmatrix} -2 & 4 \\ 3 & 8 \end{bmatrix}$, and $Z = \begin{bmatrix} 0 & 3 & 7 \\ 1 & -2 & 6 \end{bmatrix}$, calculate each of the following. If not possible, state so. SEE EXAMPLE 3

21. $X + Y$

22. $Z - X$

23. $X + Z$

24. $X - Z$

Find the additive inverse of each matrix.

SEE EXAMPLE 4

25. $Q = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

26. $R = \begin{bmatrix} 2 & 0 \\ 6 & -5 \\ -4 & 11 \end{bmatrix}$

27. $S = [4 \quad -7 \quad -8 \quad 9]$

28. $T = \begin{bmatrix} 9 & -1 \\ 4 & 10 \\ 3 & -7 \end{bmatrix}$

A segment has endpoints E (5, −1) and F (6,11).

SEE EXAMPLE 5

29. Use matrices to represent a translation of $\overline{EF}$ to $\overline{YZ}$ by 5 units right and 1 unit down. What are the coordinates of Y and Z?

30. Use matrices to represent a dilation of $\overline{EF}$ to $\overline{UV}$ by a scale factor of 4, centered at the origin. What are the coordinates of U and V?

PRACTICE & PROBLEM SOLVING

APPLY

31. **Model With Mathematics** Using a 10×10 grid, create a battleship game board with 5 ships placed. Write a matrix B for your battleship board. Use a 1 for a space a ship is placed and a 0 for a space no ship exists.

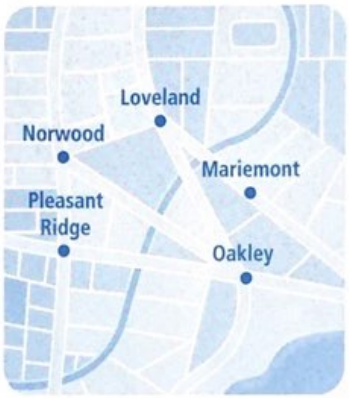
32. **Model With Mathematics** The table shows some of the men's running records in seconds.

Distance (meters)	World record	American record	Olympic record
100	9.58	9.69	9.63
200	19.19	19.32	19.30
400	43.03	43.18	43.03
1,500	206	209.3	212.07

a. Write a matrix that represents the difference between the Olympic and World records for each race distance expressed as a column matrix.

b. If all of the records in the table are expressed in seconds and are represented by a matrix B, what matrix expression could be used to convert all data to minutes?

33. **Use Structure** A matrix can be used to represent which towns are connected by a single road to each other on a map. Use a 1 to represent two towns connected to each other and a 0 to represent two towns not connected to each other. Use a 0 to show that the indicated row and column both represent the same town. Create a matrix C to represent this situation.



ASSESSMENT PRACTICE

34. Use these matrices to complete the statements.

$$A = \begin{bmatrix} 0 & 9 & 6 \\ 1 & 2 & 4 \\ 7 & -3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -7 & -2 \\ 0 & 5 & 8 \\ -3 & 1 & 1 \end{bmatrix}$$

In matrix A, the value of a_{31} is _____ the value of a_{12}. In matrix B, the value of b_{31} is _____ the value of b_{12}.

Ⓐ less than; less than

Ⓑ less than; greater than

Ⓒ greater than; less than

Ⓓ greater than; greater than

35. SAT/ACT If $5\begin{bmatrix} a \\ b \end{bmatrix} = 14\begin{bmatrix} 20 \\ 12 \end{bmatrix}$, then what is the value of $a + b$?

Ⓐ 29 Ⓑ $\frac{148}{5}$ Ⓒ $\frac{448}{5}$ Ⓓ $\frac{191}{4}$ Ⓔ $\frac{41}{5}$

30. Performance Task A computer animator uses a screen that is 1,000 pixels wide and 800 pixels tall. The animator uses matrix columns to represent three locator points on an avatar. The top row represents the horizontal coordinate of each point, and the bottom row represents the vertical coordinate. Let $P = \begin{bmatrix} 100 & 150 & 200 \\ 50 & 150 & 50 \end{bmatrix}$ represent the initial position of the avatar.

Part A The animator wants the avatar to move up at a rate of 100 pixels per second. Use addition of matrices to show the position of the avatar after 2 seconds and after 5 seconds.

Part B The animator wants the avatar to move right at a rate of 50 pixels per second. Use addition of matrices to show the position of the avatar after 3 seconds and after 8 seconds.

Part C How could the animator use scalar multiplication and matrix addition to show how the avatar moves across the screen?