Polynomial Review Stations – Answer Sheet

Station 1: Graphing Polynomials

Objective: Graph the given polynomial and analyze its end behavior and zeros.

1. **Polynomial**: $-x^3+2x^2-3x+4$
    - Degree of the polynomial: 3 (odd)
    - Leading coefficient: -1 (negative)
    - End Behavior: $x \to \infty \quad y \to -\infty$

      $x \to -\infty \quad y \to \infty$

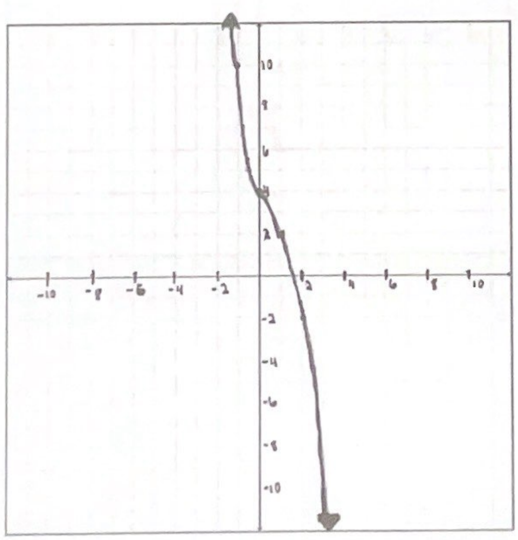
    - Zeros of the polynomial (list any real roots):

      x= 1.6506292 x= ______

      x= ______ x= ______

    - y-intercept: (0,4)

    - Sketch the graph below. Indicate zeros and end behavior. (Draw your graph on the provided grid below.)



2. **Polynomial**: $3x^4-4x^3-2x^2+x-4$

- Degree of the polynomial: 4 (even)
- Leading coefficient: 3 (positive)
- End Behavior: $x \to \infty \quad y \to \infty$

  $x \to -\infty \quad y \to \infty$

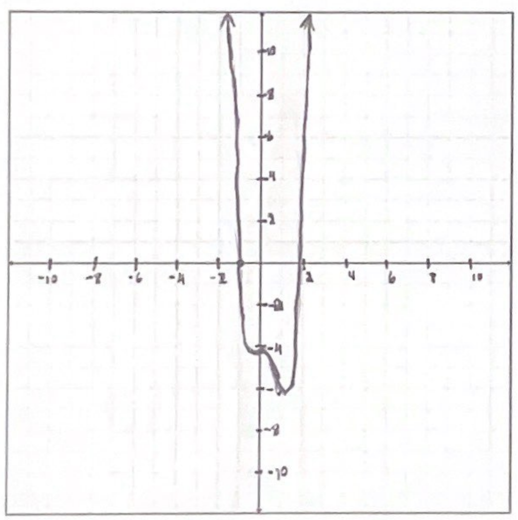
- Zeros of the polynomial (list any real roots):

  x= -1 x=

  x= 1.8201174 x=

- y-intercept: (0,-4)

- Sketch the graph below. Indicate zeros and end behavior. (Draw your graph on the provided grid below.)



3. **Polynomial:** $-4x^4+5x^3+2x^2+3x+1$

- Degree of the polynomial: 4 (even)
- Leading coefficient: -4 (negative)
- End Behavior: $x \to \infty \quad y \to -\infty$

  $x \to -\infty \quad y \to -\infty$

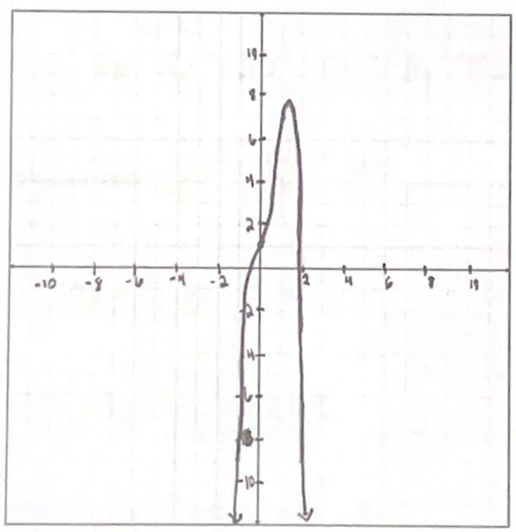
- Zeros of the polynomial (list any real roots):

  x= -0.330174 x= ________

  x= 1.8014343 x= ________

- y-intercept: (0, 1)

- Sketch the graph below. Indicate zeros and end behavior. (Draw your graph on the provided grid below.)



Station 2: Adding and Subtracting Polynomials

Objective: Combine like terms and simplify the given polynomials.

Multiply the polynomials and simplify the result.

Step-by-step Work:

1. Problem 1: $(-4x^4+14+3x^2)+(-3x^4-14x^2-8)$

   $-4x^4-3x^4+3x^2-14x^2+14-8$

   Simplified expression: $-7x^4-11x^2+6$

2. Problem 2: $(13x^2-6x^5-2x)-(-10x^2-11x^5+9x)$

   $13x^2-6x^5-2x+10x^2+11x^5-9x=-6x^5+11x^5+13x^2+10x^2-2x-9x$

   Simplified expression: $5x^5+23x^2-11x$

3. Problem 3: $(4x+2)(6x^2-x+2)$

   $24x^3-4x^2+8x+12x^2-2x+4$

   $24x^3-4x^2+12x^2+8x-2x+4$

   Simplified expression: $24x^3+8x^2+6x+4$

4. Problem 4: $(6x^2-6x-5)(7x^2+6x-5)$

$42x^4+36x^3-30x^2-42x^3-36x^2+30x-35x^2-30x+25$

$42x^4+36x^3-42x^3-30x^2-36x^2-35x^2+30x-30x+25$

Simplified expression: $42x^4-6x^3-101x^2+25$

5. Problem 5: $(3x-4y)(4x+3y)$

$12x^2+9xy-16xy-12y^2$

Simplified expression: $12x^2-7xy-12y^2$

Station 3: Polynomial Identities

Objective: Apply polynomial identities to simplify or factor the expressions.

Step-by-step Work:

1. Problem 1: $3x^3 - 24$

   $3(x^3 - 8)$

   Factored expression: $3(x-2)(x^2+2x+4)$

2. Problem 2: $x^6 + 27$

   Simplified expression: $(x^2+3)(x^4-3x^2+9)$

3. Problem 3: $2x^4 - 50$

   $2(x^4 - 25)$

   Factored expression: $2(x^2-5)(x^2+5)$

4. Problem 4: $(x-3)^5$

   $1x^5 + 5x^4(-3) + 10x^3(-3)^2 + 10x^2(-3)^3 + 5x(-3)^4 + (-3)^5$

   Simplified expression: $x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

Station 4: Dividing Polynomials

Objective: Use synthetic or long division to divide the polynomials.

Use synthetic or long division to evaluate the polynomial at the given x value.

Step-by-step Work:

1. Problem 1: $(5x^3+x^4-6x+3)\div(x+3)$

$$\begin{array}{r|rrrrr} -3 & 1 & 5 & 0 & -6 & 3 \\ & & -3 & -6 & 18 & -36 \\ \hline & 1 & 2 & -6 & 12 & \boxed{-33} \end{array}$$

Quotient: $x^3+2x^2-6x+12$ Remainder: -33

2. Problem 2: $(2x^2-17x-38)\div(2x+3)$

$$\begin{array}{r|l} & x-10 \\ 2x+3 & 2x^2-17x-38 \\ & -2x^2-3x \\ \hline & \quad -20x-38 \\ & \quad +20x+30 \\ \hline & \qquad -8 \end{array}$$

$$\begin{array}{r|rrr} -\frac{3}{2} & 2 & -17 & -38 \\ & & -3 & 30 \\ \hline & 2 & -20 & \boxed{-8} \end{array}$$

Quotient: $x-10$ Remainder: -8

3. Problem 3: $5x^4+2x^2-15x+10$ at $x=-2$

$$\begin{array}{r|rrrrr} -2 & 5 & 0 & 2 & -15 & 10 \\ & & -10 & 20 & -44 & 118 \\ \hline & 5 & -10 & 22 & -59 & \boxed{128} \end{array}$$

Answer: 128

4. Problem 4: $2x^3-11x^2+9x-20$ at $x=5$

$$\begin{array}{r|rrrr} 5 & 2 & -11 & 9 & -20 \\ & & 10 & -5 & 20 \\ \hline & 2 & -1 & 4 & \boxed{0} \end{array}$$

Answer: 0

Station 5: Zeroes and Roots of Polynomials

Objective: Use synthetic division or the Rational Root Theorem to find the zeros of the polynomial.

Step-by-step Work:

1. List all possible rational roots. $f(x) = -10x^3 - 39x^2 - 45x - 25$

   25: $\pm1, \pm5, \pm25$ 10: $\pm1, \pm2, \pm5, \pm10$

   all possible: $\pm1, \pm\frac{1}{2}, \pm\frac{1}{5}, \pm\frac{1}{10}, \pm5, \pm\frac{5}{2}, \pm25, \pm\frac{25}{2}$

   List all roots (if applicable): $\boxed{x = -\frac{5}{2}, \frac{-7+\sqrt{51}i}{10}, \frac{-7-\sqrt{51}i}{10}}$

   $$\begin{array}{r|rrrr} -\frac{5}{2} & -10 & -39 & -45 & -25 \\ & & 25 & 35 & 25 \\ \hline & -10 & -14 & -10 & \boxed{0} \end{array}$$

   $-5x^2 - 7x - 5 = 0$

   $x = \frac{7 \pm \sqrt{49 - 4(-5)(-5)}}{-10}$

2. List all possible rational roots. $f(x) = -15x^3 + 49x^2 - 55x + 25$

   25: $\pm1, \pm5, \pm25$ 15: $\pm1, \pm3, \pm5, \pm15$

   all possible: $\pm1, \pm\frac{1}{3}, \pm\frac{1}{5}, \pm\frac{1}{15}, \pm5, \pm\frac{5}{3}, \pm25, \pm\frac{25}{3}$

   List all roots (if applicable): $x = \frac{5}{3}, \frac{4+3i}{5}, \frac{4-3i}{5}$

   $$\begin{array}{r|rrrr} \frac{5}{3} & -15 & 49 & -55 & 25 \\ & & -25 & 40 & -25 \\ \hline & -15 & 24 & -15 & \boxed{0} \end{array}$$

   $-5x^2 + 8x - 5 = 0$

   $x = \frac{-8 \pm \sqrt{64 - 4(-5)(-5)}}{-10} = \frac{-8 \pm 6i}{-10}$

3. List all possible rational roots. $f(x) = 2x^3 - 7x^2 - 20x - 11$

   11: $\pm1, \pm11$ 2: $\pm1, \pm2$

   all possible: $\pm1, \pm\frac{1}{2}, \pm11, \pm\frac{11}{2}$

   List all roots (if applicable): $x = -1, \frac{11}{2}$

Station 6: Root Theorems of Polynomials

Objective: Write a polynomial function of least degree with rational coefficients that has the given zeros.

1. Write the polynomial with the given roots.

$x = -1, 1+3i, 1-3i$

$(x+1)(x-1-3i)(x-1+3i) = (x+1)((x-1)^2-(3i)^2)$

$(x+1)(x^2-2x+1+9) = (x+1)(x^2-2x+10)$

$x^3-2x^2+10x+x^2-2x+10 = x^3-x^2+8x+10$

2. Write the polynomial with the given roots.

$x = 1-i, 1+i, \sqrt{7}, -\sqrt{7}$

$(x-1+i)(x-1-i)(x-\sqrt{7})(x+\sqrt{7}) = ((x-1)^2-(i)^2)(x^2-(\sqrt{7})^2)$

$(x^2-2x+1+1)(x^2-7) = (x^2-2x+2)(x^2-7)$

$x^4-7x^2-2x^3+14x+2x^2-14 = x^4-2x^3-5x^2+14x-14$

3. Write the polynomial with the given roots.

$x = -3, 2\sqrt{2}, -2\sqrt{2}$

$(x+3)^2(x-2\sqrt{2})(x+2\sqrt{2}) = (x^2+6x+9)(x^2-(2\sqrt{2})^2) = (x^2+6x+9)(x^2-8)$

$x^4-8x^2+6x^3-48x+9x^2-72 = x^4+6x^3+x^2-48x-72$