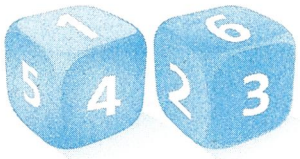
12-6 Probability and Decision Making

CRITIQUE & EXPLAIN

Your friend offers to play the following game with you. "If the product of the rolls of two number cubes is 10 or less, I win. If not, you win!"



A. If you were to play the game many times, what percent of games would you expect to win?

B. Is the game fair? Should you take the offer? Explain.

C. **Make Sense and Persevere** Suggest a way to change the game from fair to unfair, or vice versa, while still using the product of the two number cubes. Explain.

HABITS OF MIND

Use Structure Change the game from using products to using a different mathematical number operation. Can you make the game fair? Explain your reasoning.

EXAMPLE 1 Try It! Use Probability to Make Fair Decisions

1. Your trainer creates training programs for you. How can you use index cards to randomly choose the following: Strength training 1 day per week; Cardio training 2 days per week, with no consecutive days; Swimming 1 day per week.

EXAMPLE 2 Try It! Determine Whether a Decision Is Fair or Unfair

2. Justice and Tamika use the same 3 cards, but change the game. In each round, a player draws a card and replaces it, and then the other player draws. The differences between the two cards are used to score each round. Order matters, so the difference can be negative. Is each game fair? Explain.

   a. If the difference between the first and second cards is 2, Justice gets a point. Otherwise Tamika gets a point.

   b. They take turns drawing first. Each round, the first player to draw subtracts the second player's number from her own and the result is added to her total score.

EXAMPLE 3 Try It! Make a Decision Based on Expected Value

3. Additional data is collected for the TAB5000 and TAB5001. The manufacturing cost and the replacement cost for the TAB5001 remain unchanged.

   a. The production and replacement costs for the TAB5000 increased by $10. What would the expected profit be for the TAB5000?

   b. The failure rate for the TAB5001 increased by 1%. What would the expected profit be for the TAB5001?

   c. As a consultant for the company, what would you recommend they do to maximize their profit?

HABITS OF MIND

Construct Arguments When do you need to compute and compare expected values instead of just comparing probabilities? Explain.

EXAMPLE 4 Try It! Use a Binomial Distribution to Make Decisions

4. A play calls for a crowd of 12 extras with non-speaking parts. Because 10% of the extras have not shown up in the past, the director selects 15 students as extras. Find the probabilities that 12 extras show up to the performance, 15 extras show up to the performance, and more than 12 extras show up to the performance.

HABITS OF MIND

Use Structure What three expressions are multiplied together in the binomial probability formulaand what do they represent?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you use probability to make decisions?

2. **Reason** How can you use random numbers to simulate rolling a standard number cube?

3. **Error Analysis** Explain the error in Diego's reasoning.

> If a game uses random numbers, it is always fair.

4. **Use Structure** Describe what conditions are needed for a fair game.

5. **Use Appropriate Tools** Explain how you can visualize probability distributions to help you make decisions.

6. **Reason** Why must the expected value of a fair game of chance equal zero?

Do You KNOW HOW?

7. A teacher assigns each of 30 students a a unique number from 1 to 30. The teacher uses the random numbers shown to select students for presentations.
Which student was selected first? second?

randInt (1,30)	9
randInt (1,30)	9
randInt (1,30)	4

8. Three friends are at a restaurant and they all want the last slice of pizza. Identify three methods involving probability that they can use to determine who gets the last slice. Explain mathematically why each method will guarantee a fair decision.

9. Edgar rolls one number cube and Micah rolls two. If Edgar rolls a 6, he wins a prize. If Micah rolls a sum of 7, she gets a prize. Is this game fair? Explain.

10. The 10 parking spaces in the first row of the parking lot are reserved for the 12 members of the Student Council. Usually an average of ten percent of the Student Council does not drive to school dances. What is the probability that more members of the Student Council will drive to a dance than there are reserved parking spaces?

PRACTICE & PROBLEM SOLVING

UNDERSTAND

11. Reason Suppose Chris has pair of 4-sided dice, each numbered from 1 to 4, and Carolina has a pair of 10-sided dice, each numbered from 1 to 10. They decide to play a series of games against each other, using their own dice.

a. Describe a game that would be fair. Explain.

b. Describe an unfair game. Explain.

12. Construct Arguments Mr. and Ms. Mitchell have 3 children, Luke, Charlie, and Aubrey. All 3 children want to sit in the front seat. Charlie suggests that they flip a coin two times to decide who will sit in the front seat. The number of heads determines who sits in the front seat. Is this a fair method? Explain.

Number of Heads	Front Seat Passenger
0	Luke
1	Charlie
2	Aubrey

13. Error Analysis Mercedes is planning a party for 10 people. She knows from experience that about 20% of those invited will not show up. If she invites 12 people, how can she calculate the probability that more than 10 people will show up. What error did she make? What is the correct probability?

Use the binomial distribution for 12 trials, with a 20% probability, and more than 10 show up.

$(12)(0.80)^1(0.20)^{11} + (1)(0.80)^0(0.20)^{12}$ ✗

PRACTICE & PROBLEM SOLVING

PRACTICE

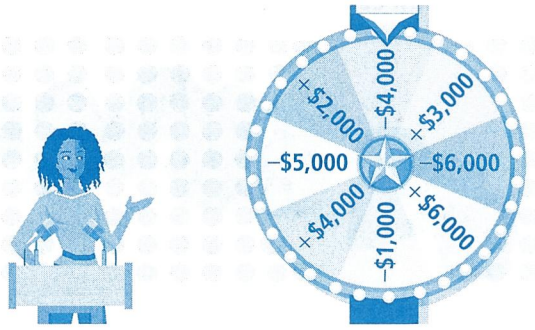
14. How can you use random integers to select 3 students from a group of 8 to serve as student body representatives, so that each student is equally likely to be selected?
SEE EXAMPLE 1

Explain whether each game is fair or unfair.
SEE EXAMPLE 2

15. When it is your turn, roll a standard number cube. If the number is even, you get a point. If it is odd, you lose a point.

16. When it is your turn, roll two standard number cubes. If the product of the numbers is even, you get a point. If the product is odd, you lose a point.

Fatima is a contestant on a game show. So far, she has won $34,000. She can keep the $34,000 or spin the spinner shown below and add or subtract the amount shown from $34,000. SEE EXAMPLE 3



17. If Fatima spins the spinner, what are her expected total winnings?

18. Would you advise Fatima to keep the $34,000 or to spin the spinner? Explain your reasoning.

19. Suppose 0.5% of people who file federal tax returns with an adjusted gross income (AGI) between $50,000 and $75,000 are audited. Of 5 people in that tax bracket for whom ABC Tax Guys prepared their taxes, 2 were audited.
SEE EXAMPLE 4

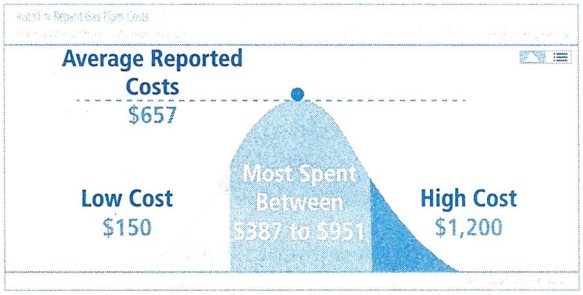
a. If 5 people with an AGI between $50,000 and $75,000 are selected at random from all the people who filed federal tax returns, what is the probability that at least 2 people are audited?

b. Would you recommend that a friend with an AGI between $50,000 and $75,000 use ABC Tax Guys to prepare her tax returns? Explain.

PRACTICE & PROBLEM SOLVING

APPLY

20. Model With Mathematics For $5.49 per month, Ms. Corchado can buy insurance to cover the cost of repairing a leak in the natural gas lines within her house. She estimates that there is a 3% chance that she will need to have such repairs made next year.



a. What is the expected cost of a gas leak, if Ms. Corchado does not buy insurance? Use the cost shown in the middle of the graph.

b. With more recent information, Ms. Corchado learns that repair costs could be as much as $1,200 dollars with an 8% probability of a leak. What is the expected cost of a gas leak with these assumptions?

c. Would you advise Ms. Corchado to buy the insurance? Explain.

21. Higher Order Thinking You are a consultant to a company that manufactures components for cell phones. One of the components the company manufactures has a 4% failure rate. Design changes have improved the quality of the component. A test of 50 of the new components found that only one of the new components is defective.

a. Before the design improvements what was the probability that among 50 of the items, at most one of the items was defective?

b. Is it reasonable to conclude that the new components have a lower failure rate than 4%?

c. Would you recommend further testing to determine whether the new parts have a lower failure rate than 4%? Explain.

ASSESSMENT PRACTICE

22. Paula, Sasha, and Yumiko live together. They want a system to determine who will wash the dinner dishes on any given night. Select all of the methods that are fair.

Ⓐ Roll a standard number cube. If the result is 1 or 2, Paula does the dishes; if 3 or 4, Sasha; if 5 or 6, Yumiko.

Ⓑ Roll a standard number cube. If the result is 1, Paula does the dishes; if 2, Sasha; if 4, Yumiko. If the result is 3, 5, or 6, roll again.

Ⓒ Roll two standard number cubes. If the sum of the numbers that come up is less than 6, Paula washes the dishes; if the sum is 8, 9, or 12, Sasha; if the sum is 6 or 7, Yumiko. If the sum is 10 or 11, roll again.

Ⓓ Write the name of each girl on a slip of paper, place the slips in a box, mix them up, and select one at random. The person whose name is selected does the dishes.

23. SAT/ACT A fair choice among a group of students may be made by flipping three coins in sequence, and noting the sequences of heads and tails. If each student is assigned one of these sequences, how many students can be selected fairly by this method?

Ⓐ 4

Ⓑ 5

Ⓒ 6

Ⓓ 7

Ⓔ 8

24. Performance Task Acme Tire Company makes two models of steel belted radial tires, Model 1001 and Model 1002.

Model	1001
Blowouts per 200,000 tires	2
Profits before any lawsuits	$60

Model	1002
Blowouts per 200,000 tires	1
Profits before any lawsuits	$56

If one of these tires fails and the company is sued, the average settlement is $1,200,000.

Part A Find the expected profit for both models of tires after any potential lawsuits. Explain.

Part B Would you recommend that the company continue selling both models? Explain.