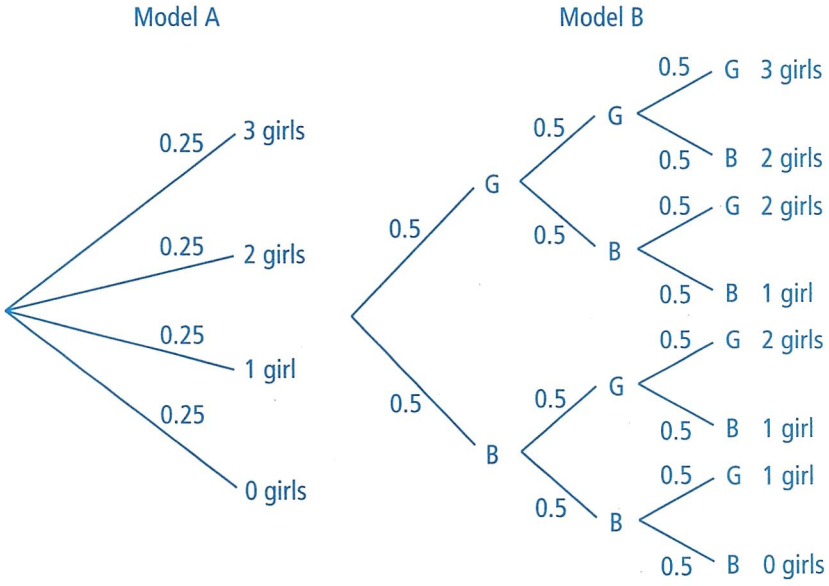
12-4 Probability Distributions

EXPLORE & REASON

Mr. and Mrs. Mason have three children. Assume that the probability of having a baby girl is 0.5 and the probability of having a baby boy is also 0.5.



A. **Reason** Which model represents the situation correctly, Model A or Model B? Explain.

B. What is the probability that Mr. and Mrs. Mason have 3 girls?

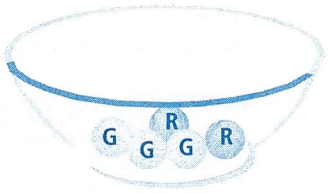
C. Compare the probability that the Masons' first child was a boy and they then had two girls to the probability that their first two children were girls and they then had a boy. Does the order affect the probabilities? Explain.

HABITS OF MIND

Look for Relationships Which combinations of children are most common? Is one order of this combination more likely? Explain.

EXAMPLE 1 Try It! Develop a Theoretical Probability Distribution

1. You select two marbles at random from the bowl. For each situation, define the theoretical probability distribution for selecting a number of red marbles on the sample space {0, 1, 2}. Is it a uniform probability distribution?



   a. You select one marble and put it back in the bowl. Then you select a second marble.

   b. You select one marble and do not put it back in the bowl. Then you select a second marble.

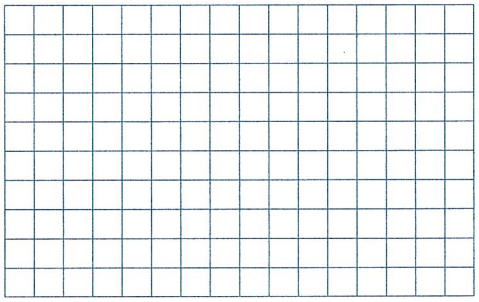
EXAMPLE 2 Try It! Develop an Experimental Probability Distribution

2. Suppose that you selected a student at random from the Drama Club and recorded the student's age.

Ages of Students in Drama Club					
Age	14	15	16	17	18
Students	4	7	10	7	9

   a. Define an experimental probability distribution on the sample space {14, 15, 16, 17, 18}.

   b. Graph the probability distribution you defined.



HABITS OF MIND

Look for Relationships Compare the graph of a uniform probability distribution with the graph of a non-uniform distribution.

EXAMPLE 3 Try It! Binomial Experiments

3. Is the experiment a binomial experiment? If so, find the probability of success. Explain.

a. You select one card at random from a set of 7 cards, 4 labeled A and 3 labeled B. Then you select another card at random from the cards that remain. For each selection, success is that the card is labeled A.

b. You roll a standard number cube 4 times. Assume that each time you roll the number cube, each number is equally likely to come up. For each roll, success is getting an even number.

EXAMPLE 4 Try It! Probabilities in a Binomial Experiment

4. To the nearest tenth of a percent, what is the probability that Terrell has more than 3 winning cards? Explain.

HABITS OF MIND

Use Structure Explain why ${}_nC_r$ appears in the formula for the probability of a binomial experiment.

Do You UNDERSTAND?

1. ESSENTIAL QUESTION What are the characteristics of a *binomial experiment*?

2. **Vocabulary** What is the difference between a binomial experiment and one that is not binomial?

3. **Error Analysis** A regular tetrahedron has four triangular sides, with one of the letters A, B, C, and D on each side. Assume that if you roll the tetrahedron, each of the letters is equally likely to end up on the bottom. {A, B, C, D} is a sample space for the experiment. Rochelle was asked to find the theoretical probability distribution for the experiment. Explain and correct the error.

$P(A) = 0.3$
$P(B) = 0.3$
$P(C) = 0.3$
$P(D) = 0.3$

Do You KNOW HOW?

Graph the probability distribution *P*.

4. Theoretical probabilities from selecting a student at random from a group of 3 students, Jack, Alani, and Seth

5. Probabilities from flipping a fair coin 3 times and counting the number of heads. The sample space is the set of numbers 0, 1, 2, 3. $P(0) = 0.125$, $P(1) = 0.375$, $P(2) = 0.375$, $P(3) = 0.125$

A bag contains 5 balls: 3 green, 1 red, and 1 yellow. You select a ball at random 4 times, replacing the ball after each selection. Calculate the theoretical probability of each event to the nearest whole percent.

6. getting a green ball exactly 3 times

7. getting a green ball exactly 4 times

8. getting a green ball at least 3 times

9. getting a yellow ball twice

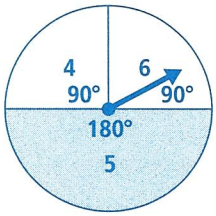
10. getting only red and green balls

PRACTICE & PROBLEM SOLVING

UNDERSTAND

11. Communicate Precisely Explain what it means for a coin to be a fair coin.

12. Reason You spin the spinner shown.



Describe a theoretical probability distribution for the experiment.

13. Communicate Precisely Five students in a class of 27 students ate hamburgers for lunch. Suppose the teacher selects a student in the class at random and then selects another student at random. Success for each selection is selecting a student who ate a hamburger. Is this a binomial experiment? Explain.

14. Error Analysis A standard number cube is rolled 7 times. Success for each roll is defined as getting a number less than 3. Abby tried to calculate the probability of 5 successes. Describe and correct her error.

$$P(5)=\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2 \approx 0.002$$ ✗

15. Mathematical Connections A marble is selected from the bowl shown 4 times. The marble is returned to the bowl after each selection.

a. Show that there are exactly ${}_4C_2$ ways to get exactly 2 green marbles.

b. How are ${}_5C_3$ and ${}_5C_2$ related? Explain.

PRACTICE & PROBLEM SOLVING

PRACTICE

A card is chosen at random from the box containing 10 cards: 3 yellow, 4 red, 2 green, and 1 blue. SEE EXAMPLES 1 AND 2

16. Define a probability distribution for this experiment on the sample space {Y, R, G, B}.

17. Graph the probability distribution.

In a certain game, the player can score 0, 1, 2, 3, or 4 points during their turn. The table shows the number of times Kennedy scored each number of points the last time she played the game. SEE EXAMPLE 2

Score	0	1	2	3	4
Frequency	3	7	9	6	5

18. Define an experimental probability distribution based on Kennedy's scores.

19. Graph the probability distribution you defined in Exercise 18.

Is the experiment a binomial experiment? Explain. SEE EXAMPLE 3

20. A quality control specialist tests 50 LED light bulbs produced in a factory. Success is that a tested light bulb burns for at least 2,000 hours without dimming. For each light bulb, the probability of success is 0.9.

21. There are 10 black and 10 red cards face down on the table. One card is selected at random. Then another card is selected at random. Success is getting a red card.

22. A basketball player is shooting 2 free throws. The probability of her making the first free throw is 0.86. The probability of her making the second free throw is 0.92.

Each time Bailey is at bat, the probability that he gets a hit is 0.250. If he bats 10 times in the course of two games, what is the probability of each result? Round to the nearest tenth of a percent. SEE EXAMPLE 4

23. He gets no hits.

24. He gets exactly 1 hit.

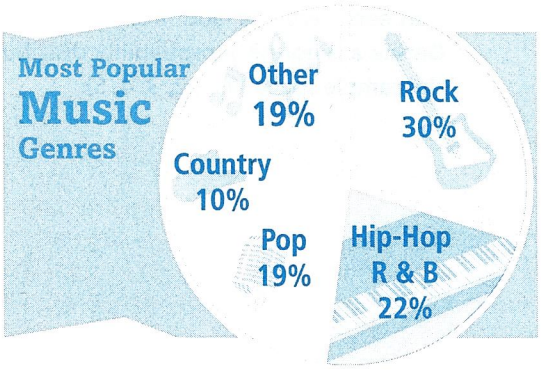
25. He gets exactly 2 hits.

26. He gets fewer than 3 hits.

PRACTICE & PROBLEM SOLVING

APPLY

27. Model with Mathematics The circle graph shows the result of a survey of the most popular types of music in the U.S., based on sales, downloads, and streaming.



a. Define a probability distribution for the sample space.

b. Graph the probability distribution.

c. According to the survey, which is the most popular type of music in the United States?

28. Higher Order Thinking A pharmaceutical company is testing a new version of a medication. In a clinical trial of the old version of the medication, 18% of the subjects taking the old medication experienced headaches.

a. Suppose that 18% of the people taking the new medications will experience headaches. If 8 subjects are selected at random and given the new medication, what is the probability that less than two of them will experience headaches?

b. Suppose that two of the eight subjects experience headaches after taking the new medication. Is that cause for concern? Explain your reasoning.

29. Communicate Precisely In a quiz show, a contestant is asked 6 questions. Each question has 5 answer choices. Assume that the contestant picks an answer at random for each question and the probability of guessing the correct answer is 20%. What is the probability of guessing correctly on at least 4 of the questions? Round your answer to the nearest tenth of a percent.

ASSESSMENT PRACTICE

30. You are going to roll a game piece two times. The game piece has 10 sides of equal area, each with one of the numbers 0 through 9. Assume that it is equally likely to land with any of the sides on top. Success is defined as getting a 3 on top.

Let P be the function defined on {0, 1, 2} such that $P(n)$ is the probability of n successes. Select all that apply.

Ⓐ This is a binomial experiment.

Ⓑ P is a probability distribution for the sample space {0, 1, 2}.

Ⓒ $P(0) = 0.81$

Ⓓ $P(1) = 0.09$

Ⓔ $P(2) = 0.01$

31. **SAT/ACT** A standard number cube is rolled 6 times. Success is defined as getting a number greater than 4. Rounded to the nearest percent, what is the probability of exactly 2 successes?

Ⓐ 2%

Ⓑ 8%

Ⓒ 23%

Ⓓ 33%

Ⓔ 50%

32. **Performance Task** Get 5 index cards. Draw a picture on one side and no picture on the other side of each card.

Part A You are going to throw all 5 cards up in the air and count the number of cards that land face up. Assume that it is equally likely that each card will land face up and face down. Define a theoretical probability distribution for the sample space {0, 1, 2, 3, 4, 5}.

Part B Perform the experiment 20 times. Each time you perform the experiment, record the number of cards that land face up. Find the experimental probability for each outcome in the sample space {0, 1, 2, 3, 4, 5} and define an experimental probability distribution the sample space.

Part C Compare the results of Part A and B. If they are different, explain why you think they are different.