12-2 Conditional Probability

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EXPLORE & REASON

At Central High School, 85% of all senior girls attended and 65% of all senior boys attended the Spring Dance. Of all attendees, 20% won a prize.

A. Assuming that the number of senior girls at Central High School is about equal to the number of senior boys, estimate the probability that a randomly selected senior won a prize at the dance. Explain.

B. **Construct Arguments** If you knew whether the selected student was a boy or a girl, would your estimate change? Explain.

HABITS OF MIND

Look for Relationships How would the probability that a senior selected at random won a prize be different if only 60% of senior girls and 50% of senior boys attended the dance? Explain.

EXAMPLE 1

Try It! Understand Conditional Probability

1. a. What is the probability that a member of the drama club is a sophomore, $P(\text{sophomore} \mid \text{drama})$?

   b. What is the probability that a sophomore is a member of the drama club, $P(\text{drama} \mid \text{sophomore})$? Is $P(\text{sophomore} \mid \text{drama})$ the same as $P(\text{drama} \mid \text{sophomore})$? Explain

EXAMPLE 2

Try It! Use the Test for Independence

2. Let R represent "the vehicle is red" and C represent "the vehicle is a car." Are the events R and C independent or dependent? Explain.

HABITS OF MIND

Make Sense and Persevere Suppose you know that events A and B are independent, and you find that $P(B \mid A) = P(A \mid B)$. What else do you know?

Notes

Assess

EXAMPLE 3

Try It! Apply the Conditional Probability Formula

3. What is the probability that a surveyed student plans to attend but is not a fan of the group?

EXAMPLE 4

Try It! Use Conditional Probability to Make a Decision

4. The marketer also has data from desktop computers. Which product is most likely to be purchased after a related search?

Computer Search and Buying Behavior (% of computer-based site visitors)

Product	Search	Search & Buy
J	35%	10%
K	28%	9%
L	26%	8%
M	24%	5%

HABITS OF MIND

Communicate Precisely Compare the formula used in Example 3, $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$, to the formula used in Example 4, $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$. How are they related? When would you use each formula?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are conditional probability and independence related in experiments?

2. **Vocabulary** How is the sample space for $P(B \mid A)$ different from the sample space for $P(B)$?

3. **Vocabulary** Why does the definition of $P(B \mid A)$ have the condition that $P(A) \neq 0$?

4. **Use Structure** Why is $P(A) \bullet P(B \mid A) = P(B) \bullet P(A \mid B)$?

5. **Error Analysis** Taylor knows that $P(R) = 0.8$, $P(B) = 0.2$, and $P(R \text{ and } B) = 0.05$. Explain Taylor's error.

$$P(B|R) = \frac{0.05}{0.2}$$
$$= 0.25$$

✗

6. **Reason** At a sports camp, a coach wants to find the probability that a soccer player is a local camper. Because 40% of the students in the camp are local, the coach reasons that the probability is 0.4. Is his conclusion justified? Explain.

Do You KNOW HOW?

7. Let $P(A) = \frac{3}{4}$, $P(B) = \frac{2}{3}$, and $P(A \text{ and } B) = \frac{1}{2}$. Find each probability.
   a. What is $P(B \mid A)$?
   b. What is $P(A \mid B)$?

8. Students randomly generate two digits from 0 to 9 to create a number between 0 and 99. Are the events "first digit 5" and "second digit 6" independent or dependent in each case? What is $P(56)$ in each experiment?
   a. The digits may not be repeated.
   b. The digits may be repeated.

9. Suppose that you select one card at random from the set of 6 cards below.

W2	B3	W4	B3	B2	W3

Let B represent the event "select a blue card" and T represent the event "select a card with a 3." Are B and T independent events? Explain your reasoning.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

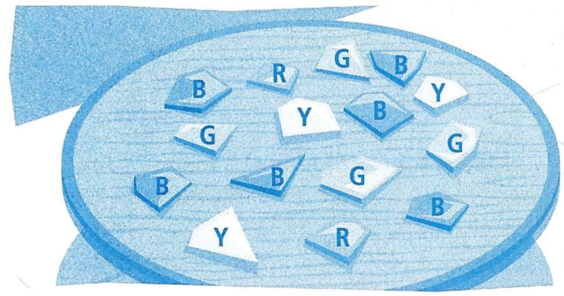
10. **Mathematical Connections** How can the formula $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ be simplified to find the probability of A and B when the events are independent? Explain.

11. **Error Analysis** From a bag containing 3 red marbles and 7 blue marbles, 2 marbles are selected without replacement. Esteban calculated the probability that two red marbles are selected. Explain Esteban's error.

$$\begin{aligned} P(\text{red}) &= 0.3 \\ P(\text{red and red}) &= P(\text{red}) \cdot P(\text{red}) \\ &= 0.3 \cdot 0.3 \\ &= 0.09 \end{aligned}$$

X

12. **Generalize** Kiyo is creating a table using mosaic tiles chosen and placed randomly. She is picking tiles without looking. How does P(yellow second | blue first) compare to P(yellow second | yellow first) if the tiles are selected without replacement? If the tiles are selected and returned to the pile because Kiyo wants a different color?



13. **Use Structure** At a fundraiser, a participant is asked to guess what is inside an unlabeled can for a possible prize. If there are two crates of cans to choose from, each having a mixture of vegetables and soup, what is the probability that the first participant will select a vegetable can from the left crate given each situation?

   a. The left crate has 2 cans of vegetables and 8 cans of soup, and the right crate has 7 cans of vegetables and 3 cans of soup.

   b. The left crate has 8 cans of vegetables and 2 cans of soup, and the right crate has 5 cans of vegetables and 5 cans of soup.

PRACTICE & PROBLEM SOLVING

PRACTICE

For Exercises 14–18, use the data in the table to find the probability of each event. SEE EXAMPLE 1

Technology Class Enrollment by Year

	Sophomore	Junior
Robotics	16	24
Game Design	18	22

14. P(Junior | Robotics)

15. P(Robotics | Junior)

16. P(Game Design | Sophomore)

17. P(Sophomore | Game Design)

18. Are year and technology class enrollment dependent or independent events? Explain. SEE EXAMPLE 2

19. At a high school, 40% of the students play an instrument. Of those students, 20% are freshmen. Of the students who do not play an instrument, 30% are freshmen. What is the probability that a student selected at random is a freshman who plays an instrument? SEE EXAMPLE 3

In a study of an experimental medication, patients were randomly assigned to take either the medication or a placebo.

Effectiveness of New Medication As Compared to a Placebo

	Medication	Placebo
Health Improved	53	47
Health Did Not Improve	65	35

20. What is the probability that a patient taking the medication showed improvement? Round to the nearest whole percent. SEE EXAMPLE 1

21. Are taking the medication and having improved health independent or dependent events? SEE EXAMPLE 2

22. Based on the data in the table, would you recommend that the medication be made available to doctors? Explain. SEE EXAMPLE 4

PRACTICE & PROBLEM SOLVING

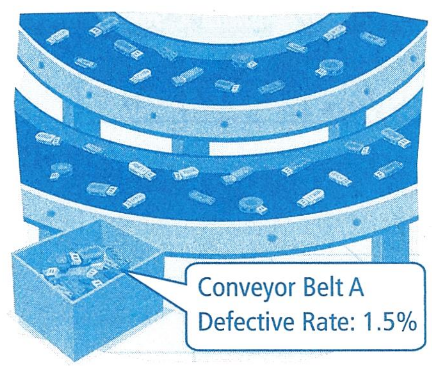
APPLY

23. Reason In a recreation center with 1,500 members, 200 are high school students. Of the members, 300 regularly swim. The 45 students of the high school swim team are all members and practice at the pool every week. What is the probability that a high school member selected at random is on the swim team?

24. Use Structure At the school fair, 5% of students will win a prize. A winner has an equally likely chance to win each prize type shown. What is the probability that a student at the fair will win a comic book? Explain.

25. Make Sense and Persevere A box contains 50 batteries, of which 10 are dead and 5 are weak. Suppose you select batteries at random from the box and set them aside for recycling if they are dead or weak. If the first battery you select is dead and the second one is weak, what is the probability that the next battery you select will be weak?

26. Higher Order Thinking An inspector at a factory has determined that 1% of the flash drives produced by the plant are defective. If assembly line A produces 20% of all the flash drives, what is the probability that a defective flash drive chosen at random is from the corresponding conveyor belt A? Explain.



ASSESSMENT PRACTICE

27. Which of the following pairs of events are independent? Select all that apply.

Ⓐ A student selected at random has a backpack. A student selected at random has brown hair.

Ⓑ Events A and B, where $P(B \mid A) = \frac{1}{3}$, $P(A) = \frac{3}{5}$ and $P(B) = \frac{5}{9}$

Ⓒ A student selected at random is a junior. A student selected at random is a freshman.

Ⓓ Events A and B, where $P(A) = 0.30$, $P(B) = 0.25$ and $P(A \text{ and } B) = 0.075$

Ⓔ Events A and B, where $P(A) = 0.40$, $P(B) = 0.3$ and $P(A \text{ and } B) = 0.012$

28. **SAT/ACT** The table shows student participation in the newspaper and yearbook by year. A student on the newspaper staff is selected at random to attend a symposium. What is the probability that the selected student is a senior?

Journalism Club Members

	Junior	Senior
Newspaper	16	9
Yearbook	8	17

Ⓐ $\frac{9}{50}$

Ⓑ $\frac{9}{26}$

Ⓒ $\frac{9}{25}$

Ⓓ $\frac{9}{17}$

Ⓔ $\frac{9}{16}$

29. **Performance Task** In a survey of 50 male and 50 female high school students, 60 students said they exercise daily. Of those students, 32 were female.

Part A Use the data to make a two-way frequency table.

Part B What is the probability that a surveyed student who exercises daily is female? What is the probability that a surveyed student who exercises regularly is male?

Part C Based on the survey, what can you conclude about the relationship between exercise and gender? Explain.