EXPLORE & REASON

A store offered customers two plans for getting bonus points:

PLAN A

10 POINTS TODAY!

Following days =
2X AS MANY POINTS
as the day before

PLAN B

1 POINT TODAY!

Following days =
3X AS MANY POINTS
as the day before

A. What expression represents the number of points received each day for Plan A?

B. What expression represents the number of points received each day for Plan B?

C. Reason On the 7th day, which plan would offer the most bonus points? Explain.

HABITS OF MIND

Reason Does Plan B always offer more points than Plan A? Explain.

EXAMPLE 1 ☑️ **Try It!** **Identify Geometric Sequences**

1. Is the sequence a geometric sequence? If so, write a recursive definition for the sequence.

   a. 1.22, 1.45, 1.68, 1.91, ...

   b. −1.5, 0.75, −0.375, 0.1875, ...

EXAMPLE 2 ☑️ **Try It!** **Translate Between Recursive and Explicit Definitions**

2. a. Given the recursive definition $a_n = \begin{cases} 12, & n = 1 \\ \frac{1}{3}a_{n-1}, & n > 1 \end{cases}$;

   what is the explicit definition for the sequence?

   b. Given the explicit definition $a_n = 6(1.2)^{n-1}$;
   what is the recursive definition?

EXAMPLE 3 ☑️ **Try It!** **Solve Problems with Geometric Sequences**

3. A geometric sequence can be used to describe the growth of bacteria in an experiment. On the first day of the experiment there were 9 bacteria in a Petri dish. On the 10th day, there are 3^{20} bacteria in the dish. How many bacteria were in the dish on the 7th day of the experiment?

HABITS OF MIND

Use Appropriate Tools How can you use the recursive definition for a geometric sequence to find the 19th term?

EXAMPLE 4 ☑ **Try It!** **Formula for the Sum of a Finite Geometric Series**

4. **a.** Write the expanded form of the series $\sum_{n=1}^{5} \frac{1}{2}(3)^{n-1}$. What is the sum?

b. Write the series $-2 + \left(\frac{-2}{3}\right) + \dots + \left(\frac{-2}{243}\right)$ using sigma notation. What is the sum?

EXAMPLE 5 ☑ **Try It!** **Find the Number of Terms in a Finite Geometric Series**

5. **a.** How many terms are in the geometric series $3 + 6 + 12 + \dots + 768$?

b. The sum of a geometric series is 155. The first term of the series is 5, and its common ratio is 2. How many terms are in the series?

EXAMPLE 6 ☑ **Try It!** **Use a Finite Geometric Series**

6. What is the monthly payment for a $40,000 loan for 4 years with an annual interest rate of 4.8%?

HABITS OF MIND

Make Sense and Persevere Why is using a formula easier than calculating and adding all 10 terms?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you represent and use geometric sequences and series?

2. **Error Analysis** Denzel claims the sequence 0, 7, 49, 343, … is a geometric sequence and the next number is 2,401. What error did he make?

3. **Vocabulary** Describe the similarities and differences between a common difference and a common ratio.

4. **Use Structure** What happens to the terms of a sequence if a_1 is positive and $r > 1$? What happens if $0 < r < 1$? Explain.

Do You KNOW HOW?

Find the common ratio and the next three terms of each geometric sequence.

5. 2, −4, 8, −16, …

6. −64, −16, −4, −1, …

7. 0.8, 2.4, 7.2, 21.6, …

8. 2, −10, 50, −250, …

9. 100, 50, 25, 12.5, …

10. In a video game, players earn 10 points for finishing the first level and twice as many points for each additional level. How many points does a player earn for finishing the fifth level? How many points will the player have earned in the game up to that point?

PRACTICE & PROBLEM SOLVING

UNDERSTAND

11. Reason True or False: If the first two terms of a geometric sequence are positive, then the third term is positive. Explain your reasoning.

12. Error Analysis The first term of a geometric sequence is 4 and grows exponentially by a factor of 3. Murphy writes out the terms and says that the sum of the 4th and 5th terms is 1,296. Explain Murphy's error and correct it.

13. Construct Arguments Write a geometric sequence with at least four terms and describe it using both an explicit and recursive definitions. How can you confirm that your sequence is geometric?

14. Higher Order Thinking Adam drops a ball from a height of 12 feet. Each bounce is 50% as high as the previous bounce. What is the total vertical distance the ball has traveled when it hits the ground for the 4th time?
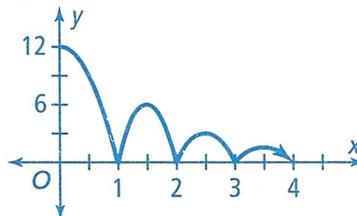
15. Model with Mathematics The Sierpinski Triangle is a fractal made by cutting an equilateral triangle into four congruent pieces and removing the center piece, leaving three smaller triangles. The process is repeated on each triangle, creating more triangles that are even smaller. Continuing this pattern, how many triangles would there be after the tenth step in the process?
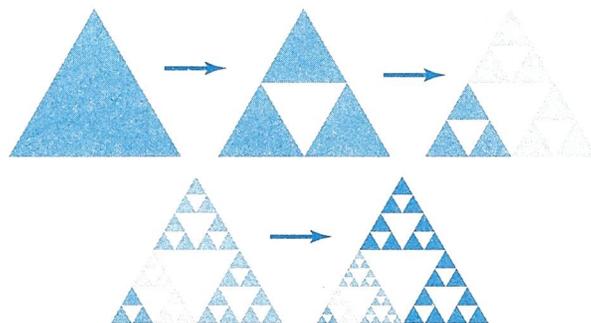
PRACTICE & PROBLEM SOLVING

PRACTICE

Is the sequence geometric? If so, write a recursive definition for the sequence. SEE EXAMPLE 1

16. 1, −3, 9, −27, …

17. 3, −15, 75, −375, …

18. 4, 5, 6, 7, …

19. 24, 8, $\frac{8}{3}$, $\frac{8}{9}$, …

20. 2, 4, 6, 8, …

21. 10, 40, 160, 640, …

Translate between the recursive and explicit definitions for each sequence. SEE EXAMPLE 2

22. $a_n = 1{,}024\left(\frac{1}{2}\right)^{n-1}$

23. $a_n = \begin{cases} 2, & n = 1 \\ -2a^{n-1}, & n > 1 \end{cases}$

24. $a_n = 35(2)^{n-1}$

25. $a_n = -6(-3)^{n-1}$

26. $a_n = \begin{cases} 1, & n = 1 \\ \frac{2}{3}a_{n-1}, & n > 1 \end{cases}$

27. In an experiment, the number of bacteria present each day form a geometric sequence. On the first day, there were 100 bacteria. On the eighth day, there were 12,800 bacteria. How many bacteria were there on the fourth day? SEE EXAMPLE 3

Write the expansion of each series. What is the sum? SEE EXAMPLE 4

28. $\sum\limits_{n=1}^{6} 4(2)^{n-1}$

29. $\sum\limits_{n=1}^{20} 6(2)^{n-1}$

30. $\sum\limits_{n=1}^{7} -4(3)^{n-1}$

31. $\sum\limits_{n=1}^{12} (-4)^{n-1}$

Write each series using sigma notation. Find the sum. SEE EXAMPLE 4

32. $8 + 16 + 32 + … + 1{,}024$

33. $-7 - 42 - 252 - … - 54{,}432$

34. $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + … + \frac{1}{80}$

35. $4 - 12 + 36 - … + 2{,}916$

36. The sum of a geometric series is 31.75. The first term of the series is 16, and its common ratio is 0.5. How many terms are in the series?
SEE EXAMPLE 5

37. What is the monthly payment for a $12,000 loan for 7 years with an annual interest rate of 2.7%? SEE EXAMPLE 6

PRACTICE & PROBLEM SOLVING

APPLY

38. **Model With Mathematics** Kelley opens a bank account to save for a down payment on a car. Her initial deposit is $250, and she plans to deposit 10% more each month. Kelley's goal is to have $2,000 in the account after six months. Will she meet her goal?

39. **Make Sense and Persevere** Henry just started his own cleaning business. He is using word-of-mouth from his current clients to promote his business. He currently has seven clients.

    a. Five of his clients really like Henry's work and each told two friends the following month. This group each told two friends the following month, and so on for a total of five months. Assuming no one heard twice, how many people have had or heard of a positive experience with Henry's cleaning business?

b. The two unhappy clients each told five people the following month. This group each told five people, and so on, for five months. Assuming no one heard twice, how many people have had or heard of a negative experience with Henry's cleaning business?

40. **Model with Mathematics** Ricardo bought a motorcycle for $15,000. The value depreciates 15% at the start of every year. What is the value of the motorcycle after three years?

Time	Value
0 Yr	$15,000
1 Yr	
2 Yr	
3 Yr	
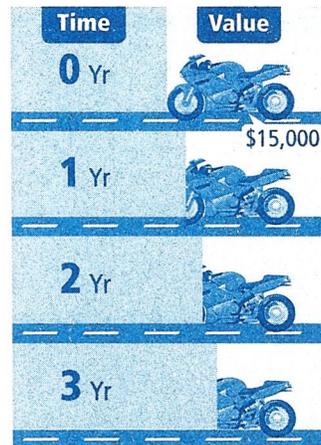
41. The first term of a geometric series is −1, and the common ratio is −2. Fill in the number to complete the sentence.

If the sum of the series is −43, there are _____?_____ terms in the series.

42. SAT/ACT What is the value of the 11th term in the following geometric sequence?

$$\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \cdots$$

Ⓐ 3^4

Ⓑ 3^5

Ⓒ 3^6

Ⓓ 3^7

Ⓔ 3^8

43. Performance Task An avid collector wants to purchase a signed basketball from a particular playoff game. He plans to put away 4% more money each year, in a safe at his home, to save up for the basketball. In the sixth year, he puts $580 in the safe and realizes that he has exactly enough money to purchase the basketball.

Price = Year 5 savings + $580.00
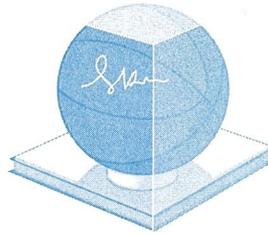
Part A How much money did the collector put into the safe the first year?

Part B To the nearest dollar, how much did the collector pay for the signed playoff basketball?