EXPLORE & REASON

Look at the graph of $y = \log x$ and the ordered pairs shown.
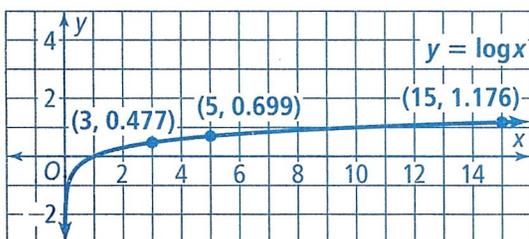
A. Complete the table shown.

x	3	5	15
log x			

B. Look for Relationships What is the relationship between the numbers 3, 5, and 15? What is the relationship between the logarithms of 3, 5, and 15?

C. What is your prediction for the value of log 45? log 75? Explain.

HABITS OF MIND

Generalize Do you think that the relationships you found in the Explore & Reason activity would also hold for natural logarithms? Give an example.

EXAMPLE 1   ☑️ **Try It!** **Prove a Property of Logarithms**

1. Prove the Quotient Property of Logarithms.

EXAMPLE 2   ☑️ **Try It!** **Expand Logarithmic Expressions**

2. Use the properties of logarithms to expand each expression.

a. $\log_7\left(\dfrac{r^3 t^4}{v}\right)$

b. $\ln\left(\dfrac{7}{225}\right)$

EXAMPLE 3   ☑️ **Try It!** **Write Expressions as Single Logarithms**

3. Write each expression written as a single logarithm.

a. $5\log_2 c - 7\log_2 n$

b. $2\ln 7 + \ln 2$

HABITS OF MIND

Make Sense and Persevere Using the fact that $\log 2 \approx 0.3010$ and $\log 3 \approx 0.4771$, what is $\log 18$? Show how you know.

EXAMPLE 4 ☑ **Try It!** **Apply Properties of Logarithms**

4. What is the concentration of hydrogen ions in a liter of orange juice?

HABITS OF MIND

Generalize What types of numbers have logarithms that are negative? Explain.

EXAMPLE 5 ☑ **Try It!** **Evaluate Logarithmic Expressions by Changing the Base**

5. Estimate the value of each logarithm. Then use a calculator to find the value of each logarithm to the nearest thousandth.

a. $\log_2 7$ 　　　　　　　　　　　　b. $\log_5 3$

EXAMPLE 6 ☑ **Try It!** **Use the Change of Base Formula**

6. What is the solution to the equation $3^x = 15$? Express the solution as a logarithm and then evaluate. Round to the nearest thousandth.

HABITS OF MIND

Use Appropriate Tools Why is the Change of Base Formula useful when evaluating a logarithm with a calculator?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are the properties of logarithms used to simplify expressions and solve logarithmic equations?

2. **Vocabulary** While it is not necessary to change to base 10 when applying the Change of Base Formula, why is it common to do so?

3. **Error Analysis** Amanda claimed the expanded form of the expression $\log_4(c^2 d^5)$ is $5\log_4 c + 5\log_4 d$. Explain the error Amanda made.

Do You KNOW HOW?

4. Use the properties of logarithms to expand the expression $\log_6\left(\frac{49}{5}\right)$.

5. Use the properties of logarithms to write the expression $5 \ln s + 6 \ln t$ as a single logarithm.

6. Use the formula $pH = \log\frac{1}{[H^+]}$ to write an expression for the concentration of hydrogen ions, $[H^+]$, in a container of baking soda with a pH of 8.9.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

7. **Use Structure** Without applying the Change of Base Formula, explain how to use $\log_3 2 \approx 0.631$ and $\log_3 5 \approx 1.465$ to approximate $\log_3\left(\frac{2}{5}\right)$.

8. **Communicate Precisely** Explain what is meant by *expanding a logarithmic expression*. How are the processes of *expanding logarithmic expressions* and *writing logarithmic expressions as a single logarithm* related?

9. **Higher Order Thinking** The graph of $y = \log\left(\frac{1}{x}\right)$ and $y = -\log x$ are shown. Notice the graph is the same for both equations. Use properties of logarithms to explain why the graphs are the same.
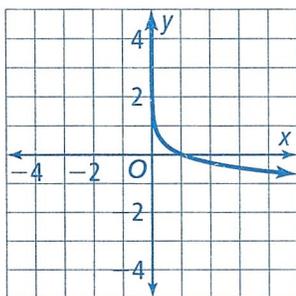
10. **Communicate Precisely** Emma used the Change of Base Formula to solve the equation $6^x = 72$ and found that $x = 2.387$. How can Emma check her solution?

11. **Error Analysis** Describe and correct the error a student made in writing the logarithmic expression in terms of a single logarithm.

$$\log_3 2 + \tfrac{1}{2}\log_3 y = \log_3 2y^2 \quad \textcolor{blue}{\boldsymbol{\times}}$$

12. **Error Analysis** A student wants to approximate $\log_2 9$ with her calculator. She enters the equivalent expression $\frac{\ln 2}{\ln 9}$, but the decimal value is not close to her estimate of 3. What happened?

$$\log_2 9 = \frac{\ln 2}{\ln 9} \quad \textcolor{blue}{\boldsymbol{\times}}$$

PRACTICE & PROBLEM SOLVING

PRACTICE

13. Use the properties of exponents to prove the Power Property of Logarithms. SEE EXAMPLE 1

Use the properties of logarithms to expand each expression. SEE EXAMPLE 2

14. $\log_5\left(\frac{2}{3}\right)$

15. $\log_6(2m^5n^3)$

16. $\ln 2x^5$

17. $\log_2\left(\frac{x}{5y}\right)$

Use the properties of logarithms to write each expression as a single logarithm. SEE EXAMPLE 3

18. $9\ln x - 6\ln y$

19. $\log_5 6 + \frac{1}{2}\log_5 y$

20. $2\log 10 + 4\log(3x)$

21. $\frac{1}{3}\ln 27 - 3\ln(2y)$

22. $8\log_3 2 + 5\log_3 c + 7\log_3 d$

23. Use properties of logarithms to show that $pH = \log\frac{1}{[H^+]}$ can be written as $pH = -\log[H^+]$.
SEE EXAMPLE 4

Use the Change of Base Formula to evaluate each logarithm. Round to the nearest thousandth.
SEE EXAMPLE 5

24. $\log_4 9$

25. $\log_6 5$

26. $\ln 3$

27. $\log_2 7$

28. $\log_9 12$

29. $\ln 23$

Use the Change of Base Formula to solve each equation for x. Give an exact solution as a logarithm and an approximate solution rounded to the nearest thousandth. SEE EXAMPLE 6

30. $3^x = 4$

31. $5^x = 11$

32. $8^x = 10$

33. $2^x = 30$

34. $7^x = 100$

35. $4^x = 55$

PRACTICE & PROBLEM SOLVING

APPLY

36. **Make Sense and Persevere** The loudness of sound is measured in decibels. For a sound with intensity I (in watts per square meter), its loudness $L(I)$ (in decibels) is modeled by the function $L(I) = 10 \log \frac{I}{I_0}$, where I_0 represents the intensity of a barely audible sound (approximately 10^{-12} watts per square meter).

Heavy Traffic: intensity of $I = 10^4$ watts per square meter

Music through headphones: 40 dB
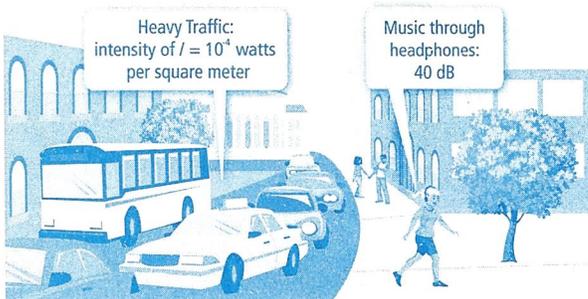
a. Find the decibel level of the sound made by the heavy traffic.

b. Find the intensity of the sound that is made by music playing at 40 decibels.

c. How many times as great is the intensity of the traffic than the intensity of the music?

37. **Model with Mathematics** Miguel collected data on the attendance at an amusement park and the daily high temperature. He found that the model $A = 2 \log t + \log 5$ approximated the attendance A, in thousands of people, at the amusement park, when the daily high temperature is t degrees Fahrenheit.

a. Use properties of logarithms to simplify Miguel's formula.

b. The daily high temperatures for the week are below.

Mon	Tue	Wed	Thu	Fri
87° 66°	73° 67°	65° 61°	72° 57°	80° 59°

Daily temperatures show highs and lows in °F.

What is the expected attendance on Wednesday? Round to the nearest person.

ASSESSMENT PRACTICE

38. Match each expression with an equivalent expression.

   I. $\log_4 20$

   II. $2\log_2 5$

   III. $\log_2 5$

   VI. $4\log_4 2$

   Ⓐ $\log_2 20 - \log_2 4$

   Ⓑ $\log_4 2 + \log_4 10$

   Ⓒ $\dfrac{\log 25}{\log 2}$

   Ⓓ $\log_2 4$

39. SAT/ACT Use the properties of logarithms to write the following expression in terms of a single logarithm.

$$2(\log_3 20 - \log_3 4) + 0.5 \log_3 4$$

  Ⓐ $\log_3 4$

  Ⓑ $\log_3 5$

  Ⓒ $\log_3 25$

  Ⓓ $\log_3 50$

40. Performance Task The magnitude, or intensity, of an earthquake is measured on the Richter scale. For an earthquake where the amplitude of its seismographic trace is A, its magnitude is modeled by the function:

$$R(A) = \log\frac{A}{A_0},$$

where A_0 represents the amplitude of the smallest detectable earthquake.

Part A An earthquake occurs with an amplitude 200 times greater than the amplitude of the smallest detectable earthquake, A_0. What is the magnitude of this earthquake on the Richter scale?

Part B Approximately how many times as great is the amplitude of an earthquake measuring 6.8 on the Richter scale than the amplitude of an earthquake measuring 5.9 on the Richter scale?

Part C Suppose the intensity of one earthquake is 150 times as great as that of another. How much greater is the magnitude of the more intense earthquake than the less intense earthquake?