CRITIQUE & EXPLAIN

Earthquakes make seismic waves through the ground. The equation $y = 10^x$ relates the height, or amplitude, in microns, of a seismic wave, y, and the power, or magnitude, x, of the ground-shaking it can cause.

Magnitude, x	Amplitude, y
2	100
3	1,000
?	4,500
4	10,000

Taylor and Chen used different methods to find the magnitude of the earthquake with amplitude 5,500.

Taylor

5,500 is halfway between 1,000 and 10,000.

3.5 is halfway between 3 and 4.

The magnitude is about 3.5.

Chen

$y = 10^x$

$10^3 = 1,000$
$10^4 = 10,000$
$10^{3.5} \approx 3,162$
$10^{3.7} \approx 5,012$
$10^{3.8} \approx 6,310$
$10^{3.74} \approx 5,500$

The magnitude is about 3.74.

A. What is the magnitude of an earthquake with amplitude 100,000? How do you know?

B. **Construct Arguments** Critique Taylor's and Chen's work. Is each method valid? Could either method be improved?

C. Describe how to express the exact value of the desired magnitude.

HABITS OF MIND

Reason Taylor reasoned that since 5,500 was halfway between 1,000 and 10,000, that the magnitude had to be halfway between 3 and 4. What is incorrect about Taylor's reasoning?

EXAMPLE 1 ✓ **Try It!** **Understand Logarithms**

1. Write the logarithmic form of $y = 8^x$.

EXAMPLE 2 ✓ **Try It!** **Convert Between Exponential and Logarithmic Forms**

2. a. What is the logarithmic form of $7^3 = 343$?

b. What is the exponential form of $\log_4 16 = 2$?

HABITS OF MIND

Communicate Precisely Write a sentence to describe what the equation $\log_a b = c$ means.

EXAMPLE 3 ✓ **Try It!** **Evaluate Logarithms**

3. What is the value of each logarithmic expression?

a. $\log_3 \left(\dfrac{1}{81} \right)$ **b.** $\log_7 (-7)$ **c.** $\log_5 5^9$

EXAMPLE 4 ☑️ **Try It!** **Evaluate Common and Natural Logarithms**

4. What is the value of each logarithmic expression to the nearest ten-thousandth?

    a. log 321   **b.** ln 1,215   **c.** log 0.17

HABITS OF MIND

Reason In order for log x or ln x to be defined, what must be true about x?

EXAMPLE 5 ☑️ **Try It!** **Solve Equations With Logarithms**

5. Solve each equation. Round to the nearest thousandth.

    a. log $(3x - 2) = 2$ **b.** $e^{x+2} = 8$

EXAMPLE 6 ☑️ **Try It!** **Use Logarithms to Solve Equations**

6. What is the magnitude of an earthquake with a seismic energy of 1.8×10^{23} joules?

HABITS OF MIND

Make Sense and Persevere How do logarithms help you to solve an equation in which the variable is an exponent?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What are logarithms and how are they evaluated?

2. **Error Analysis** Amir said the expression $\log_5 (-25)$ simplifies to -2. Explain Amir's possible error.

3. **Vocabulary** Explain the difference between the common logarithm and the natural logarithm.

4. **Make Sense and Persevere** How can logarithms help to solve an equation such as $10^t = 656$?

Do You KNOW HOW?

Write each equation in logarithmic form.

5. $2^{-6} = \frac{1}{64}$

6. $e^4 \approx 54.6$

Write each equation in exponential form.

7. $\log 200 \approx 2.301$

8. $\ln 25 \approx 3.22$

Evaluate the expression.

9. $\log_4 64$

10. $\log \frac{1}{100}$

11. $\ln e^5$

12. Solve for x. $4e^x = 7$.
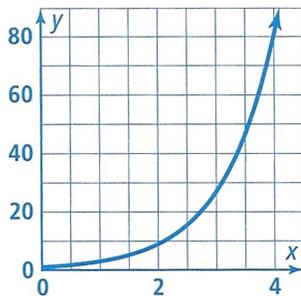
PRACTICE & PROBLEM SOLVING

UNDERSTAND

13. **Make Sense and Persevere** If the LN button on your calculator were broken, how could you still use your calculator to find the value of the expression ln 65?

14. **Error Analysis** Describe and correct the error a student made in solving an exponential equation.

$$16e^t = 98$$
$$e^t = 6.125$$
$$6.125t = \ln e$$
$$t = \frac{\ln e}{6.125}$$ ✗

15. **Higher Order Thinking** Use the graph of $y = 3^x$ to estimate the value of $\log_3 50$. Explain your reasoning.

16. **Generalize** For what values of x is the expression $\log_4 x < 0$ true?

17. **Use Structure** A student says that $\log_3\left(\frac{1}{27}\right)$ simplifies to -3. Is the student correct? Explain.

18. **Use Structure** Explain why the expression ln 1,000 is not equal to 3.

PRACTICE & PROBLEM SOLVING

PRACTICE

Write the inverse of each exponential function.
SEE EXAMPLE 1

19. $y = 4^x$

20. $y = 10^x$

21. $y = 7^x$

22. $y = a^x$

Write each equation in logarithmic form.
SEE EXAMPLE 2

23. $3^8 = 6{,}561$

24. $e^{-3} \approx 0.0498$

25. $5^0 = 1$

26. $7^3 = 343$

Write each equation in exponential form.
SEE EXAMPLE 2

27. $\log \frac{1}{100} = -2$

28. $\log_8 64 = 2$

29. $\ln 148.41 \approx 5$

30. $\log_2 \frac{1}{32} = -5$

Evaluate each logarithmic expression. SEE EXAMPLE 3

31. $\log_5 \frac{1}{125}$

32. $\log_6 (-216)$

33. $\log_3 3^4$

34. $\log_2 32$

35. $\log_9 729$

36. $\log_8 \frac{1}{64}$

37. $\log_7 0$

38. $\log_7 7^a$

Use a calculator to evaluate each expression. Round to the nearest ten-thousandth. SEE EXAMPLE 4

39. $\log 78.5$

40. $\log 0.24$

41. $\ln(-37)$

42. $\ln 41.5$

43. $\log 12$

44. $\ln 3$

Solve each equation. Round answers to the nearest ten-thousandth. SEE EXAMPLES 5 AND 6

45. $\log(7x + 6) = 3$

46. $2.75e^t = 38.6$

47. $\ln(3x - 1) = 2$

48. $10^{t+1} = 50$

49. $1.5e^t = 27$

50. $\log(x - 3) = -1$

51. How long does it take for \$250 to grow to \$600 at 4% annual percentage rate compounded continuously? Round to the nearest year.

✏️ PRACTICE & PROBLEM SOLVING

APPLY

52. Model with Mathematics Michael invests $1,000 in an account that earns a 4.75% annual percentage rate compounded continuously. Peter invests $1,200 in an account that earns a 4.25% annual percentage rate compounded continuously. Which person's account will grow to $1,800 first?

53. Reason The Richter magnitude of an earthquake is $R = 0.67\log(0.37E) + 1.46$, where E is the energy (in kilowatt-hours) released by the earthquake.

At a richter magnitude of 4 and above, the walls in your house may start to crack.

Not felt   Extreme

1 2 3 ④ 5 6 7 8 9 10
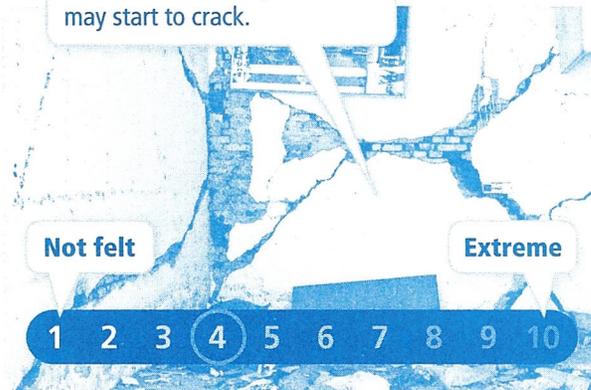
a. What is the magnitude of an earthquake that releases 11,800,000,000 kilowatt-hours of energy? Round to the nearest tenth.

b. How many kilowatt-hours of energy would an earthquake have to release in order to be an 8.2 on the Richter scale? Round to the nearest whole number.

c. What number of kilowatt-hours of energy would an earthquake have to release in order for walls to crack? Round to the nearest whole number.

54. Reason The function $c(t) = 108e^{-0.08t} + 75$ calculates the temperature, in degrees Fahrenheit, of a cup of coffee that was handed out a drive-thru window t minutes ago.

a. What is the temperature of the coffee in the instant that it is handed out the window?

b. After how many minutes is the coffee in the cup 98 degrees Fahrenheit? Round to the nearest whole minute.

✓ **ASSESSMENT PRACTICE**

55. Given that $\log_b x < 0$, which of the following are true? Select all that apply.

Ⓐ $b < 0$

Ⓑ $x < 0$

Ⓒ $b > 0$

Ⓓ $x > 0$

Ⓔ $x < 1$

56. SAT/ACT In the equation $\log_3 a = b$, if b is a whole number, which of the following CANNOT be a value for a?

Ⓐ 1

Ⓑ 3

Ⓒ 6

Ⓓ 9

Ⓔ 81

57. Performance Task Money is deposited into two separate accounts. The money in one account is compounded continuously. The money in the other account is not compounded continuously. Neither account has any money withdrawn in the first 6 years.

Year	Account 1 Balance ($)	Account 2 Balance ($)
0	400	500
1	433.31	575
2	469.40	650
3	508.50	725
4	550.85	800
5	596.72	875

Part A Write a function to calculate the amount of money in each account given t, the number of years since the account was opened. Describe the growth in each account.

Part B Will the amount of money in Account 1 ever exceed the amount of money in Account 2? Explain. If so, when will that occur?