

## EXPLORE & REASON

Juan is studying exponential growth of bacteria cultures. Each is carefully controlled to maintain a specific growth rate. Copy and complete the table to find the number of bacteria cells in each culture.

| Culture | Initial Number of Bacteria | Growth Rate per Day | Time (days) | Final Number of Bacteria |
|---------|----------------------------|---------------------|-------------|--------------------------|
| A       | 10,000                     | 8%                  | 1           |                          |
| B       | 10,000                     | 4%                  | 2           |                          |
| C       | 10,000                     | 2%                  | 4           |                          |
| D       | 10,000                     | 1%                  | 8           |                          |

A. What is the relationship between the daily growth rate and the time in days for each culture?

B. **Look for Relationships** Would you expect a culture with a growth rate of  $\frac{1}{2}\%$  and a time of 16 days to have more or fewer cells than the others in the table? Explain.

### HABITS OF MIND

**Model With Mathematics** Describe another situation that you could represent using an exponential function.

**EXAMPLE 1** **Try It! Rewrite an Exponential Function to Identify a Rate**

1. The population in a small town is increasing annually at 1.8%. What is the quarterly rate of population increase?

**HABITS OF MIND**

**Generalize** Why can't you just divide an annual interest rate by 4 to obtain a quarterly interest rate?

**EXAMPLE 2** **Try It! Understand Continuously Compounded Interest**

2. \$3,000 is invested in an account that earns 3% annual interest, compounded monthly.
  - a. What is the value of the account after 10 years?
  - b. What is the value of the account after 100 years?

**EXAMPLE 3** **Try It! Understanding Continuously Compounded Interest**

3. If you continued the table for  $n = 1,000,000$ , would the value in the account increase or decrease? How do you know?

**HABITS OF MIND**

**Generalize** Which yields the greatest return on investment: compounding quarterly, hourly, or continuously? Explain.



**EXAMPLE 4****Try It! Find Continuously Compounded Interest**

4. You invest \$125,000 in an account that earns 4.75% annual interest, compounded continuously.
- What is the value of the account after 15 years?
  - What is the value of the account after 30 years?

**EXAMPLE 5****Try It! Use Two Points to Find an Exponential Model**

5. A surveyor determined the value of an area of land over a period of several years since 1950. The land was worth \$31,000 in 1954 and \$35,000 in 1955. Use the data to determine an exponential model that describes the value of the land.

**EXAMPLE 6****Try It! Use Regression to Find an Exponential Model**

6. According to the model in Example 6, what was the approximate temperature 35 minutes after cooling started?

**HABITS OF MIND**

**Generalize** How can a graph help you determine whether an exponential model is appropriate for a data set? Explain.

## Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** Why do you develop exponential models to represent and interpret situations?

2. **Error Analysis** The exponential model  $y = 5,000(1.05)^t$  represents the amount Yori earns in an account after  $t$  years when \$5,000 is invested. Yori said the monthly interest rate of the exponential model is 5%. Explain Yori's error.

3. **Vocabulary** Explain the similarities and differences between compound interest and continuously compounded interest.

4. **Communicate Precisely** Kylee is using a calculator to find an exponential regression model. How would you explain to Kylee what the variables in the model  $y = ab^x$  represent?

## Do You KNOW HOW?

The exponential function models the annual rate of increase. Find the monthly and quarterly rates.

5.  $f(t) = 2,000(1.03)^t$

6.  $f(t) = 500(1.055)^t$

Find the total amount of money invested in an account at the end of the given time period.

7. compounded monthly,  $P = \$2,000$ ,  $r = 3\%$ ,  $t = 5$  years

8. continuously compounded,  $P = \$1,500$ ,  $r = 1.5\%$ ,  $t = 6$  years

Write an exponential model given two points.

9. (3, 55) and (4, 70)

10. (7, 12) and (8, 25)

11. Paul invests \$6,450 in an account that earns continuously compounded interest at an annual rate of 2.8%. What is the value of the account after 8 years?



## PRACTICE & PROBLEM SOLVING

### UNDERSTAND

12. **Error Analysis** Suppose \$6,500 is invested in an account that earns interest at a rate of 2% compounded quarterly for 10 years. Describe and correct the error a student made when finding the value of the account.

$$A = 6500 \left( 1 + \frac{0.02}{12} \right)^{12(10)}$$

$$A = 7937.80$$

X

13. **Communicate Precisely** The points (2, 54.61) and (4, 403.48) are points on the graph of an exponential model in the form  $y = a \cdot e^x$ .

- Explain how to write the exponential model, and then write the model.
- How can you use the exponential model to find the value of  $y$  when  $x = 8$ ?

14. **Model with Mathematics** Use the points listed in the table for years 7 and 8 to find an exponential model. Then use a calculator to find an exponential model for the data. Explain how to find each model. Predict the amount in the account after 15 years.

| Time (yr) | Amount (\$) |
|-----------|-------------|
| 1         | 3,225       |
| 2         | 3,500       |
| 3         | 3,754       |
| 4         | 4,042       |
| 5         | 4,368       |
| 6         | 4,702       |
| 7         | 5,063       |
| 8         | 5,456       |

15. **Higher Order Thinking** A power model is a type of function in the form  $y = a \cdot x^b$ . Use the points (1, 4), (2, 8), (3, 16) and (4, 64) and a calculator to find an exponential model and a power model for the data. Then use each model to predict the value of  $y$  when  $x = 6$ . Graph the points and models in the same window. What do you notice?

## PRACTICE & PROBLEM SOLVING

### PRACTICE

Find the amount in the account for the given principal, interest rate, time, and compounding period. SEE EXAMPLES 2 AND 4

16.  $P = 800$ ,  $r = 6\%$ ,  $t = 9$  years; compounded quarterly
17.  $P = 3,750$ ,  $r = 3.5\%$ ,  $t = 20$  years; compounded monthly
18.  $P = 2,400$ ,  $r = 5.25\%$ ,  $t = 12$  years; compounded semi-annually
19.  $P = 1,500$ ,  $r = 4.5\%$ ,  $t = 3$  years; compounded daily
20.  $P = \$1,000$ ,  $r = 2.8\%$ ,  $t = 5$  years; compounded continuously
21.  $P = \$16,000$ ,  $r = 4\%$ ,  $t = 25$  years; compounded continuously

Write an exponential model given two points.

SEE EXAMPLE 5

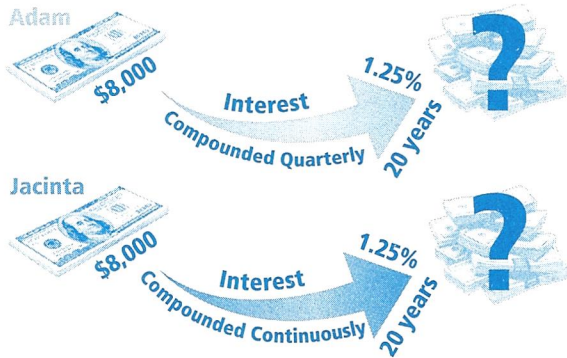
22. (9, 140) and (10, 250)
23. (6, 85) and (7, 92)
24. (10, 43) and (11, 67)
25. In 2012, the population of a small town was 3,560. The population is decreasing at a rate of 1.7% per year. How can you rewrite an exponential growth function to find the quarterly decay rate? SEE EXAMPLE 1
26. Selena took a pizza out of the oven and it started to cool to room temperature (68°F). She will serve the pizza when it reaches 150°F. She took the pizza out of the oven at 5:00 P.M. When can she serve it? SEE EXAMPLE 6

| Time (min) | Temperature (°F) |
|------------|------------------|
| 5          | 310              |
| 8          | 264              |
| 10         | 238              |
| 15         | 202              |
| 20         | 186              |
| 25         | 175              |

## PRACTICE & PROBLEM SOLVING

### APPLY

- 27. Reason** Adam invests \$8,000 in an account that earns 1.25% interest, compounded quarterly for 20 years. On the same date, Jacinta invests \$8,000 in an account that earns continuous compounded interest at a rate of 1.25% for 20 years. Who do you predict will have more money in their account after 20 years? Explain your reasoning.



- 28. Make Sense and Persevere** A blogger found that the number of visits to her Web site increases 5.6% annually. The Web site had 80,000 visits this year. Write an exponential model to represent this situation. By what percent does the number of visits increase daily? Explain how you found the daily rate.

- 29. Use Structure** Jae invested \$3,500 at a rate of 2.25% compounded continuously in 2010. How much will be in the account in 2025? How much interest will the account have earned by 2025?

- 30. Model with Mathematics** A scientist is conducting an experiment with a pesticide. Use a calculator to find an exponential model for the data in the table. Use the model to determine how much pesticide remains after 180 days.

|       |        |
|-------|--------|
| Day 0 | 20.00g |
| Day 1 | 14.73g |
| Day 2 | 11.29g |
| Day 3 | 8.38g  |
| Day 4 | 6.82g  |
| Day 5 | 4.75g  |
| Day 6 | 3.15g  |



### ASSESSMENT PRACTICE

31. The table shows the account information of five investors. Which of the following are true, assuming no withdrawals are made? Select all that apply.

| Employee | $P$  | $r$  | $t(\text{years})$ | Compound      |
|----------|------|------|-------------------|---------------|
| Anna     | 4000 | 1.5% | 12                | Quarterly     |
| Nick     | 2500 | 3%   | 8                 | Monthly       |
| Lori     | 7200 | 5%   | 15                | Annually      |
| Tara     | 2100 | 4.5% | 6                 | Continuously  |
| Steve    | 3800 | 3.5% | 20                | Semi-annually |

- Ⓐ After 12 years, Anna will have about \$4,788.33 in her account.
- Ⓑ After 8 years, Nick will have about \$3,177.17 in his account.
- Ⓒ After 15 years, Lori will have about \$15,218.67 in her account.
- Ⓓ After 6 years, Tara will have about \$2,750.93 in her account.
- Ⓔ After 20 years, Steve will have about \$7,629.00 in his account.

32. **SAT/ACT** Rick invested money in a continuous compound account with an interest rate of 3%. How long will it take Rick's account to double?

- Ⓐ about 2 years
- Ⓑ about 10 years
- Ⓒ about 23 years
- Ⓓ about 46 years
- Ⓔ about 67 years

33. **Performance Task** Cassie is financing a \$2,400 treadmill. She is going to use her credit card for the purchase. Her card charges 17.5% interest compounded monthly. She is not required to make minimum monthly payments.

**Part A** How much will Cassie pay in interest if she waits a full year before paying the full balance?

**Part B** How much additional interest will Cassie pay if she waits two full years before paying the full balance?

**Part C** If both answers represent a single year of interest, why is the answer in B greater than the answer in A?