



Activity



EXPLORE & REASON

Each number path will lead you from a number in the domain, the set of all real numbers, to a number in the range.

Number Path $f: x \rightarrow f(x)$

- Start with x .
- Subtract 3.
- Multiply by -2 .
- Add 5.

Number Path $g: x \rightarrow g(x)$

- Start with x .
- Add 1.
- Square the value.
- Subtract 2.

- A. Follow the number paths to find $f(1)$ and $g(1)$.
- B. Identify all possible values of x that lead to $f(x) = 7$ and all values that lead to $g(x) = 7$.
- C. **Communicate Precisely** Based on the two number paths, under what conditions can you follow a path back to a unique value in the domain?

HABITS OF MIND

Model With Mathematics Write a rule for Number Path f . Write a rule for the process of following the number path backward. How do the two rules compare?

5-6

Inverse Relations and Functions



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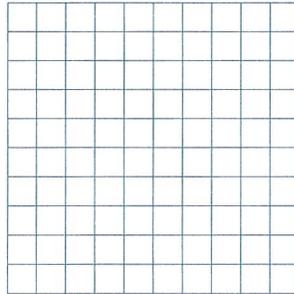
EXAMPLE 1  **Try It!** Represent the Inverse of a Relation

1. Identify the inverse relation. Is it a function?

| | | | | | | |
|-----|----|---|---|---|---|----|
| x | -1 | 0 | 1 | 2 | 3 | 4 |
| y | 9 | 7 | 5 | 3 | 1 | -1 |

EXAMPLE 2  **Try It!** Find an Equation of an Inverse Relation

2. Let $f(x) = 2x + 1$.
- a. Write an equation to represent the inverse of f .
- b. How can you use the graph of f to determine if the inverse of f is a function? Explain your answer.

**HABITS OF MIND**

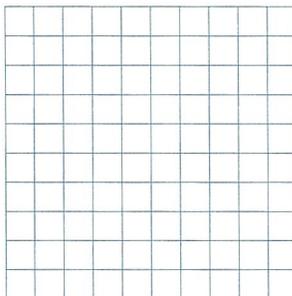
Communicate Precisely Think of $f(x) = 2x + 1$ as a number path: start with x , multiply by 2, and add 1. How could you describe the path from the result back to x ?

EXAMPLE 3  **Try It!** Restrict a Domain to Produce an Inverse Function

3. Find the inverse of each function by identifying an appropriate restriction of its domain.
- a. $f(x) = x^2 + 8x + 16$ b. $f(x) = x^2 - 9$

**EXAMPLE 4** **Try It! Find an Equation of an Inverse Function**

4. Let $f(x) = 2 - \sqrt[3]{x+1}$.

a. Sketch the graph of f .b. Verify that the inverse will be a function and write an equation for $f^{-1}(x)$.**EXAMPLE 5** **Try It! Use Composition to Verify Inverse Functions**5. Use composition to determine whether f and g are inverse functions.

a. $f(x) = \frac{1}{4}x + 7$, $g(x) = 4x - 7$

b. $f(x) = \sqrt[3]{x-1}$, $g(x) = x^3 + 1$

HABITS OF MIND

Construct Arguments Dana says that the functions $f(x) = (x-2)^2 + 5$ and $g(x) = \sqrt{x-5} + 2$ are inverses. Keegan says that the functions are inverses only if the domain is restricted. Is either person correct? Explain.

EXAMPLE 6 **Try It! Rewrite a Formula**6. The manufacturer of a gift box designs a box with length and width each twice as long as its height. Find a formula that gives the height h of the box in terms of its volume V . Then give the length of the box if the volume is 640 cm^3 .**HABITS OF MIND**

Make Sense and Persevere In the formula $V = \frac{4}{3}\pi r^3$, which variable is the dependent variable? In the formula $r = \sqrt[3]{\frac{3}{4\pi}V}$, which variable is the dependent variable?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you find the inverse of a function and verify the two functions are inverses?

2. **Error Analysis** Abi said the inverse of $f(x) = 3x + 1$ is $f^{-1}(x) = \frac{1}{3}x - 1$. Is she correct? Explain.

3. **Construct Arguments** Is the inverse of a function always a function? Explain.

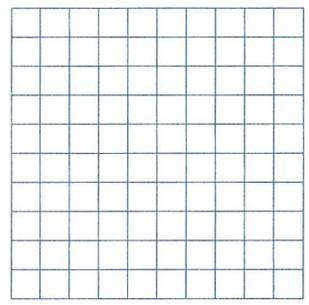
Do You KNOW HOW?

Consider the function $f(x) = -\frac{1}{2}x + 5$.

4. Write an equation for the inverse of $f(x)$.

5. Use composition to show that $f(x)$ and the equation you wrote are inverses.

6. Sketch a graph of f and its inverse.



7. How can you verify by the graph of f and its inverse that they are indeed inverses?

8. Is the inverse of $f(x)$ a function? Explain.

 **PRACTICE & PROBLEM SOLVING**
UNDERSTAND

9. **Reason** Explain how to find the range of the inverse of $f(x) = \sqrt{2x - 3}$ without finding $f^{-1}(x)$.

10. **Error Analysis** Describe and correct the error a student made in finding the inverse of the function $f(x) = x^2 - 4$.

$$\begin{aligned}
 f(x) &= x^2 - 4 \\
 x &= y^2 - 4 \\
 \sqrt{x} &= \sqrt{y^2 - 4} \\
 \sqrt{x} &= y - 2 \\
 \sqrt{x} + 2 &= y \\
 f^{-1}(x) &= \sqrt{x} + 2 \quad \mathbf{X}
 \end{aligned}$$

11. **Higher Order Thinking** What is the inverse operation of raising a number to the 4th power? How can you use the inverse operation of a number raised to the 4th power to find the inverse of the function $f(x) = x^4 - 1$? Is the inverse of f a function? Explain.

12. **Communicate Precisely** A function has the ordered pairs $(1, 3)$, $(7, 4)$, $(8, 6)$, and $(9, y)$. What restrictions are there on the value of y so that the inverse of the function is also a function? Explain.

13. **Construct Arguments** What is the inverse of the function $a(b) = \frac{1}{4}b^2$? Show how to use composition of functions to prove you found the correct inverse.

14. **Construct Arguments** A relation has one element in its domain and two elements in its range. Is the relation a function? Is the inverse of the relation a function? Explain.

15. **Mathematical Connections** Find the x - and y -intercepts of the function $y = 2x + 1$. What are the intercepts of the inverse function? How are the intercepts related?

PRACTICE & PROBLEM SOLVING

PRACTICE

Identify the inverse relation. Is it a function?

SEE EXAMPLE 1

16.

| | | | | | | |
|---|----|----|----|---|----|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 9 | 3 | -4 | 8 | -6 | 3 |

17.

| | | | | | | |
|---|----|---|---|----|---|---|
| x | -2 | 1 | 0 | 1 | 2 | 3 |
| y | -7 | 6 | 8 | -1 | 3 | 7 |

Write an equation to represent the inverse of f . Sketch the graphs of f , $y = x$, and the inverse of f on the same coordinate axes. Is the inverse of f a function? SEE EXAMPLE 2

18. Let $f(x) = x + 3$.

19. Let $f(x) = 4x - 1$.

20. Let $f(x) = x^2 + 1$.

21. Let $f(x) = \sqrt{x + 5}$.

Find the inverse of the function by identifying an appropriate restriction of its domain. SEE EXAMPLE 3

22. $f(x) = x^2 + 4x + 4$

23. $f(x) = x^2 - 6x + 9$

24. $f(x) = x^2 - 2$

25. $f(x) = x^2 + 5$

Find an equation of the inverse function, and state the domain of the inverse. SEE EXAMPLE 4

26. $f(x) = 2x^2 - 5$

27. $f(x) = \sqrt{x + 6}$

28. $f(x) = 3x + 10$

29. $f(x) = \sqrt{x - 9}$

Use composition to determine whether f and g are inverse functions. SEE EXAMPLE 5

30. $f(x) = 2x - 9$, $g(x) = \frac{1}{2}x + 9$

31. $f(x) = \sqrt{\frac{x+4}{3}}$, $g(x) = 3x^2 - 4$

32. A manager purchased cones for ice cream. Find a formula for the length of the radius, r , of a cone in terms of its volume, V . Then find the length of the radius of a cone if the volume is $290\pi \text{ cm}^3$ and the height is 15 cm. SEE EXAMPLE 6



PRACTICE & PROBLEM SOLVING

APPLY

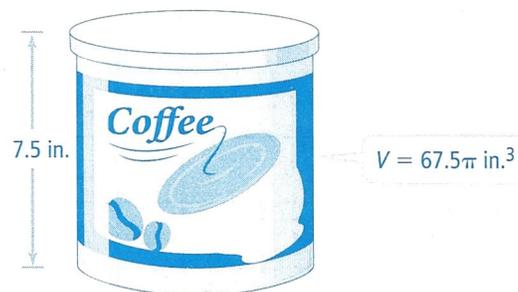
33. **Model With Mathematics** The formula for converting Celsius to Fahrenheit is $F = \frac{5}{9}(C - 32)$. Find the inverse formula, and use it to find the Celsius temperature when the Fahrenheit temperature is 56° F.

34. **Reason** A DJ charges an hourly fee and an equipment setup fee.



- Write a function for the cost, C , of hiring a DJ for n hours.
- Find the inverse of the cost function. What does the function represent?
- If the DJ charged \$550, for how many hours was she hired? Use the inverse function.

35. **Reason** A coffee can is in the shape of a cylinder.



- Find the formula that gives the radius of the coffee can r in terms of the volume V and height h .
- Describe any restrictions on the formula.
- What is the radius of a coffee can given the volume is 67.5π in.³ and the height is 7.5 in.?

 **ASSESSMENT PRACTICE**

36. Choose Yes or No to tell whether each function has an inverse that is a function.

| | Yes | No |
|----------------------------|--------------------------|--------------------------|
| a. $f(x) = 2x - 9$ | <input type="checkbox"/> | <input type="checkbox"/> |
| b. $f(x) = x^2 + 4$ | <input type="checkbox"/> | <input type="checkbox"/> |
| c. $f(x) = x^3 - 6$ | <input type="checkbox"/> | <input type="checkbox"/> |
| d. $f(x) = \sqrt{2x + 7}$ | <input type="checkbox"/> | <input type="checkbox"/> |
| e. $f(x) = x^2 - 10x + 25$ | <input type="checkbox"/> | <input type="checkbox"/> |

37. **SAT/ACT** What is the range of the inverse of $f(x) = \sqrt{-ax + b} - c$, where a , b , and c are real numbers?

- Ⓐ $y \geq \frac{a}{b}$
- Ⓑ $y \leq \frac{b}{a}$
- Ⓒ $y \geq -\frac{a}{b}$
- Ⓓ $y \geq -\frac{b}{a}$
- Ⓔ $y \geq c$

38. **Performance Task** The table shows several functions and some of the inverses of those functions. The table also shows whether some of the inverses are functions.

| Function | Inverse | Is the inverse a function? |
|--------------|---------------------------|----------------------------|
| $f(x) = x$ | $f^{-1}(x) = x$ | yes |
| $g(x) = x^2$ | $g^{-1}(x) = \pm\sqrt{x}$ | no |
| $h(x) = x^3$ | $h^{-1}(x) = \sqrt[3]{x}$ | yes |
| $k(x) = x^4$ | | |
| $m(x) = x^5$ | | |
| $n(x) = x^6$ | | |

Part A Determine the inverses of the remaining functions in the table.

Part B Determine if the inverses of the remaining functions in the table are functions.

Part C Make a conjecture about the power of a function if the inverse of that function is a function.