



## MODEL & DISCUSS

In business, the term *profit* is used to describe the difference between the money the business earns (revenue) and the money the business spends (cost).

- A. Grooming USA charges \$25 for every pet that is groomed. Let  $x$  represent the number of pets groomed in a month. Define a revenue function for the business.



- B. Materials and labor for each pet groomed costs \$15. The business also has fixed costs of \$1,000 each month. Define a cost function for this business.

- C. Last month, Grooming USA groomed 95 pets. Did they earn a profit? What would the profit be if the business groomed 110 pets in a month?

- D. **Generalize** Explain your procedure for calculating the profit for Grooming USA. Suppose you wanted to calculate the profit for several different scenarios. How could you simplify your process?

### HABITS OF MIND

**Make Sense and Persevere** A business “breaks even” when its revenue equals its costs. How many pets would Grooming USA have to groom in order to break even?







## Do You UNDERSTAND?

- ESSENTIAL QUESTION** How do you combine, multiply, divide, and compose functions, and how do you find the domain of the resulting function?
- Vocabulary** In your own words, define and provide an example of a composite function.
- Error Analysis** Reagan said the domain of  $\frac{f}{g}$  when  $f(x) = 5x^2$  and  $g(x) = x + 3$  is the set of real numbers. Explain why Reagan is incorrect.
- Use Structure** Explain why changing the order in which two functions occur affects the result when subtracting and dividing the functions.

## Do You KNOW HOW?

Let  $f(x) = 3x^2 + 5x + 1$  and  $g(x) = 2x - 1$ .

5. Identify the rule for  $f + g$ .

6. Identify the rule for  $f - g$ .

7. Identify the rule for  $g - f$ .

Let  $f(x) = x^2 + 2x + 1$  and  $g(x) = x - 4$ .

8. Identify the rule for  $f \cdot g$ .

9. Identify the rule for  $\frac{f}{g}$ , and state the domain.

10. Identify the rule for  $\frac{g}{f}$ , and state the domain.

11. If  $f(x) = 2x^2 + 5$  and  $g(x) = -3x$ , what is  $f(g(x))$ ?

 **PRACTICE & PROBLEM SOLVING**
**UNDERSTAND**

**12. Generalize** Does  $f \circ g$  always equal  $g \circ f$ ? Justify your response.

**13. Construct Arguments** Explain why the domain for the quotient of functions might not be the set of all real numbers.

**14. Error Analysis** Describe and correct the error a student made in finding the rule for the composition  $f \circ g$  of the functions  $f(x) = 3x^2 - x + 2$  and  $g(x) = 2x + 1$ .

$$\begin{aligned}
 f \circ g &= f(g(x)) \\
 &= 3(2x + 1)^2 - 2x + 1 + 2 \\
 &= 3(4x^2 + 4x + 1) - 2x + 1 + 2 \\
 &= 12x^2 + 12x + 3 - 2x + 1 + 2 \\
 &= 12x^2 + 10x + 6
 \end{aligned}$$



**15. Make Sense and Persevere** Identify the rules for two functions,  $f(x)$  and  $g(x)$ , for which  $f \circ g = g \circ f$ .

**16. Higher Order Thinking** Suppose two functions,  $f(x)$  and  $g(x)$  are only defined by the ordered pairs listed below.

$$f = (6, 7), (5, 2), (4, 1), (10, 8)$$

$$g = (5, 4), (3, 6), (1, 5), (2, 10)$$

Find the ordered pairs that comprise  $(f \circ g)(x)$ .

**17. Mathematical Connections** How is the process of finding the rule for the composition of functions related to the order of operations in arithmetic?

**18. Make Sense and Persevere** Recalling that the identity function is  $f(x) = x$ , identify the rules for two functions  $f(x)$  and  $g(x)$ , for which  $f(g(x)) = x$ .

**19. Construct Arguments** Is it possible that the result of subtracting two linear functions is a horizontal line? If so, give an example. What must be true about the two linear functions? If not, explain why it is not possible.

## PRACTICE & PROBLEM SOLVING

### PRACTICE

Let  $f(x) = 2x^2 + 5x - 1$  and  $g(x) = 3x + 2$ . Identify the rules for the following functions. SEE EXAMPLE 1

20.  $f + g$

21.  $f - g$

22. Suppose the demand  $d$ , in units sold, for a company's jeans at price  $x$ , in dollars, is  $d(x) = 600 - 4x$ .



a. If revenue = price  $\times$  demand, write the rule for the function  $r(x)$ , which represent the company's expected revenue in jean sales. Then state the domain of this function.

b. If the price is \$40, how much revenue will the company earn? SEE EXAMPLE 2

23. Identify the rule and domain for  $\frac{f}{g}$  when  $f(x) = x^2 + 3x - 28$  and  $g(x) = x + 7$ .

SEE EXAMPLE 3

Let  $f(x) = 4x - 5$  and  $g(x) = -7x$ . Evaluate each expression. SEE EXAMPLE 4

24.  $f(g(3))$

25.  $f(g(x))$

26.  $g(f(2))$

27.  $g(f(x))$

Let  $f(x) = x^2 + x$  and  $g(x) = 9 - 2x$ . Identify the rules for the following functions. SEE EXAMPLE 5

28.  $f \circ g$

29.  $g \circ f$

30. A sporting goods store is running a summer sale on its snowboards. Kayden is interested in a snowboard that normally costs \$400. The store is offering a \$50 instant rebate, as well as a 10% discount.

In which order should these special offers be applied to the cost of the snowboard in order to benefit Kayden? Explain. SEE EXAMPLE 6



## PRACTICE & PROBLEM SOLVING

### APPLY

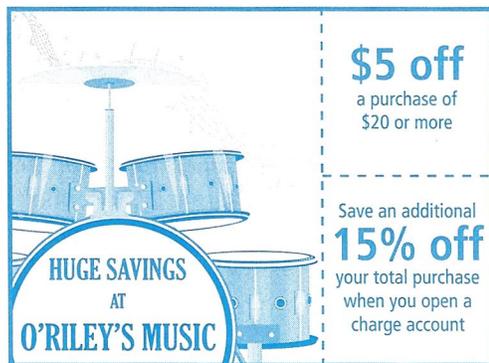
**31. Model With Mathematics** The cost (in dollars) to produce  $x$  shovels in a factory is given by the function  $C(x) = 20x + 500$ . The number of shovels that can be produced in  $h$  hours is given by the function  $x(h) = 30h$ .

a. Find the rule for  $C(x(h))$ .

b. Find the cost when  $h = 8$  hours.

c. Explain what the answer to part (b) represents.

**32. Use Structure** A music store is running the following promotions.



**HUGE SAVINGS**  
AT  
**O'RILEY'S MUSIC**

**\$5 off**  
a purchase of  
\$20 or more

Save an additional  
**15% off**  
your total purchase  
when you open a  
charge account

a. Use composition of functions to find the sale price of a \$90 purchase when the \$5 off discount is applied prior to the 15% off discount.

b. Use composition of functions to find the sale price of a \$90 purchase when the 15% off discount is applied prior to the \$5 off discount.

c. In which order is the deal better for the consumer? Explain.

**33. Reason** From 2000 to 2015, the number of births,  $b$ , (in the hundreds) in Fairfield County can be modeled by the function  $b(x) = 300 - 5x$ . The number of deaths,  $d$ , (in the hundreds) can be modeled by the function  $d(x) = 10x + 5$ . The variable  $x$  represents the number of years since 2000.

a. Which function operation can be used to represent the net increase in the population?

b. Write and simplify a function which represents the net increase in the population,  $p$ , against  $x$ , the number of years since 2000. State the domain of this function.

**ASSESSMENT PRACTICE**

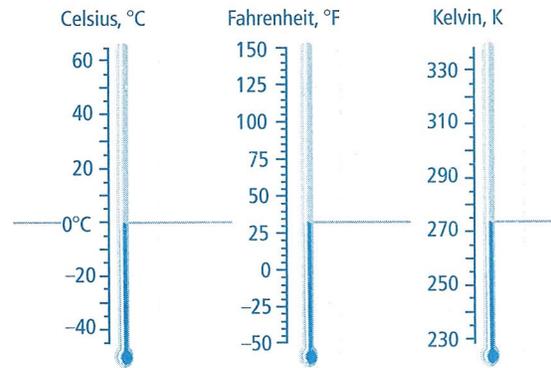
34. Given that  $f(x) = x^2 + 8x + 3$  and  $g(x) = -x - 7$ , which of the following are true? Select all that apply.

- (A)  $f + g = x^2 + 7x - 4$
- (B)  $f(g(x)) = x^2 + 6x - 4$
- (C) The domain of  $\frac{f}{g}$  is the set of all real numbers.
- (D)  $f(x) \cdot g(x) = -x^3 - 15x^2 + 53x + 21$
- (E) In the composition  $g \circ f$ , the output  $f(x)$  is used as the input for  $g$ .

35. **SAT/ACT** Find the value of  $f(g(5))$  if  $f(x) = 4x + 1$  and  $g(x) = x^2 + 6$ .

- (A) 101
- (B) 124
- (C) 125
- (D) 676
- (E) 682

36. **Performance Task** The temperature in degrees Celsius is 32 less than the Fahrenheit temperature, multiplied by five ninths. The temperature in degrees Kelvin is the number of degrees Celsius plus 273.



**Part A** Derive a conversion formula for finding the number of degrees Kelvin, given the temperature in Fahrenheit.

**Part B** Using your conversion formula from part (a), find the temperature in degrees Kelvin when the temperature is 27°F. Round to the nearest whole number if necessary.