



Activity

# 5-4

## Solving Radical Equations



PearsonRealize.com

### EXPLORE & REASON

A. Solve  $3(a + 1)^2 + 2 = 11$ . Use at least two different methods.

B. Try each of the methods you used in part (a) to solve  $\sqrt[3]{(a + 1)} + 2 = 11$ .

C. **Generalize** Which of the methods is better suited for solving an equation with a radical? What problems arise when using the other method?

#### HABITS OF MIND

**Construct Arguments** The squares of two numbers are equal. Does that mean that the two numbers themselves must also be equal? Explain.

**EXAMPLE 1**  **Try It! Solve an Equation With One Radical**

1. Solve each radical equation.

a.  $\sqrt{x-2} + 3 = 5$

b.  $\sqrt[3]{x-1} = 2$

**EXAMPLE 2**  **Try It! Rewrite a Formula**2. The speed,  $v$ , of a vehicle in relation to its stopping distance,  $d$ , is represented by the equation  $v = 3.57\sqrt{d}$ . What is the equation for the stopping distance in terms of the vehicle's speed?**HABITS OF MIND****Reason** Reese solved the equation in **1(a)** by first squaring both sides. Is this an appropriate first step? Why or why not?**EXAMPLE 3**  **Try It! Identify an Extraneous Solution**

3. Solve each radical equation. Identify any extraneous solutions.

a.  $x = \sqrt{7x+8}$

b.  $x+2 = \sqrt{x+2}$

**EXAMPLE 4** **Try It!** Solve Equations With Rational Exponents

4. Solve each equation.

a.  $(x^2 - 3x - 6)^{\frac{3}{2}} - 14 = -6$

b.  $(x + 8)^2 = (x - 10)^{\frac{5}{2}}$

**EXAMPLE 5** **Try It!** Solve an Equation With Two Radicals

5. Solve each radical equation. Check for extraneous solutions.

a.  $\sqrt{x + 4} - \sqrt{3x} = -2$

b.  $\sqrt{15 - x} - \sqrt{6x} = -3$

**HABITS OF MIND****Reason** Why are extraneous solutions a possibility for radical equations?**EXAMPLE 6** **Try It!** Solve a Radical Inequality

6. A doctor calculates that a particular dose of medicine is appropriate for an individual whose BSA is less than 1.8. If the mass of the individual is 75 kg, how many centimeters tall can he or she be for the dose to be appropriate?

**HABITS OF MIND****Make Sense and Persevere** How are the steps for solving a radical inequality different from the steps for solving a radical equation?

## Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you solve equations that include radicals or rational exponents?

2. **Construct Arguments** How can you use a graph to show that the solution to  $\sqrt[3]{84x + 8} = 8$  is 6?

3. **Vocabulary** Why does solving a radical equation sometimes result in an extraneous solution?

4. **Error Analysis** Neil said that  $-3$  and  $6$  are the solutions to  $\sqrt{3x + 18} = x$ . What error did Neil make?

5. **Communicate Precisely** Describe how you would solve the equation  $x^{\frac{2}{3}} = n$ . How is this solution method to be interpreted if the equation had been written in radical form instead?

## Do You KNOW HOW?

Solve for  $x$ .

6.  $3\sqrt{x + 22} = 21$

7.  $\sqrt[3]{5x} = 25$

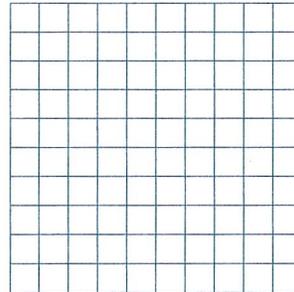
In exercises 8 and 9, find the extraneous solution.

8.  $\sqrt{8x + 9} = x$

9.  $x = \sqrt{24 - 2x}$

10. Rewrite the equation  $y = \sqrt{\frac{x - 48}{6}}$  to isolate  $x$ .

11. Use a graph to find the solution to the equation  $9 = \sqrt{3x + 11}$ .



Solve each equation.

12.  $(3x + 2)^{\frac{2}{5}} = 4$

13.  $\sqrt{2x - 5} - \sqrt{x - 3} = 1$

14.  $\sqrt{x + 2} + \sqrt{3x + 4} = 2$

## PRACTICE & PROBLEM SOLVING

### UNDERSTAND

**15. Generalize** Explain how to identify an extraneous solution for an equation containing a radical expression.

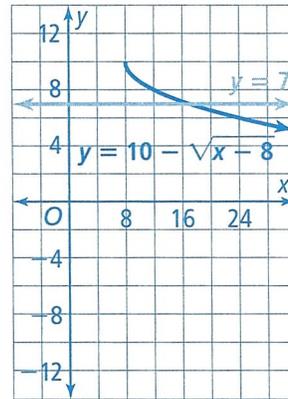
**16. Look for Relationships** Write a radical equation that relates a square's perimeter to its area. Explain your reasoning. Use  $s$  to represent the side length of the square.

**17. Error Analysis** Describe and correct the error a student made in rewriting the equation to isolate  $y$ .

$$\begin{aligned}
 x &= \frac{\sqrt{58 + y}}{1.98} \\
 1.98x &= \sqrt{58 + y} \\
 1.98x^2 &= 58 + y \\
 1.98x^2 - 58 &= y
 \end{aligned}$$

X

**18. Use Appropriate Tools** Use the equations represented in the graph below to find the point of intersection.



**19. Higher Order Thinking** Some, but not all, equations with rational exponents have extraneous solutions. What is the relationship between the exponents and the possibility of having extraneous solutions for equations with rational exponents? Explain your reasoning.

**20. Communicate Precisely** Describe the process used to solve an equation with two radical expressions. How is this process different from solving an equation with only one radical expression?

# PRACTICE & PROBLEM SOLVING

## PRACTICE

Solve each radical equation. SEE EXAMPLE 1

21.  $\sqrt[3]{x} + 8 = 13$

22.  $\sqrt{4x} = 11$

23.  $\sqrt{75 + x} - 6 = 14$

24.  $25 - \sqrt[4]{x} = 22$

Solve for  $y$ . SEE EXAMPLE 2

25.  $x = 3(\sqrt[3]{15 + y})$

26.  $x = \frac{\sqrt{2y}}{26}$

27.  $x = \frac{\sqrt{y - 14.2}}{0.05}$

28.  $x = \frac{1}{3}(\sqrt[4]{y})$

Solve each radical equation. Check for extraneous solutions. SEE EXAMPLE 3

29.  $x = \sqrt{x + 6}$

30.  $2x = \sqrt{17x - 15}$

31.  $4x = \sqrt{6x + 10}$

32.  $x = \sqrt{56 - x}$

Solve. SEE EXAMPLE 4

33.  $0.5(x^2 + 5x + 136)^{\frac{2}{3}} = 50$

34.  $2(x^2 - 12x - 4)^{\frac{1}{2}} - 3 = 15$

35.  $(x^2 + 4x + 5)^{\frac{3}{2}} + 1 = 0$

Solve each radical equation. Check for extraneous solutions. SEE EXAMPLE 5

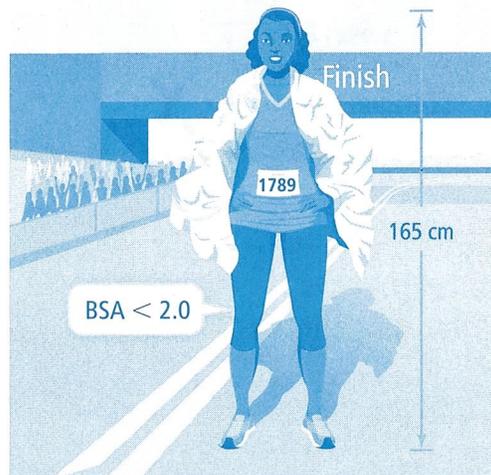
36.  $\sqrt{6 + x} - \sqrt{x - 5} = 2$

37.  $\sqrt{4x + 5} - \sqrt{x + 1} = 1$

38.  $\sqrt{x + 1} + 1 = \sqrt{x + 3}$

Solve using the formula  $BSA = \sqrt{\frac{H \cdot M}{3,600}}$ .  
SEE EXAMPLE 6

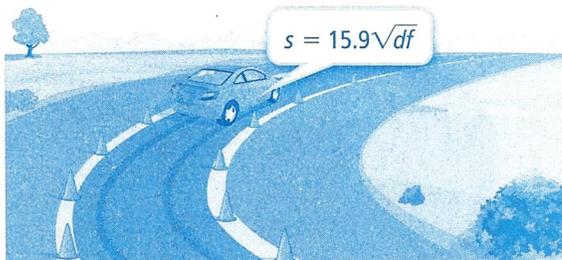
39. A sports medicine specialist determines that a hot-weather training strategy is appropriate for a 165 cm tall individual whose BSA is less than 2.0. To the nearest hundredth, what can the mass of the individual be for the training strategy to be appropriate?



## PRACTICE & PROBLEM SOLVING

### APPLY

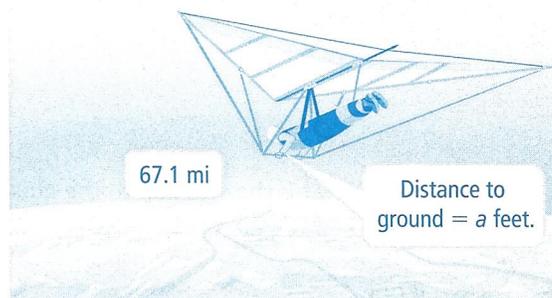
40. **Use Structure** Specialists can determine the speed a vehicle was traveling from the length of its skid marks,  $d$ , and the coefficient of friction,  $f$ . The formula for calculating the speed,  $s$ , is  $s = 15.9\sqrt{df}$ . Rewrite the formula to solve for the length of the skid marks.



41. **Make Sense and Persevere** The half-life of a certain type of soft drink is 5 h. If you drink 50 mL of this drink, the formula  $y = 50(0.5)^{\frac{t}{5}}$  tells the amount of the drink left in your system after  $t$  hours. How much of the soft drink will be left in your system after 16 hours?

42. **Model With Mathematics** Big Ben's pendulum takes 4 s to swing back and forth. The formula  $t = 2\pi\sqrt{\frac{L}{32}}$  gives the swing time,  $t$ , in seconds, based on the length of the pendulum,  $L$ , in feet. What is the minimum length necessary to build a clock with a pendulum that takes longer than Big Ben's pendulum to swing back and forth?

43. **Make Sense and Persevere** Derek is hang gliding on a clear day at an altitude of  $a$  feet. His visibility,  $v$ , is 67.1 mi. Use the formula  $v = 1.225\sqrt{a}$  to find the altitude at which Derek is hang gliding.



**ASSESSMENT PRACTICE**

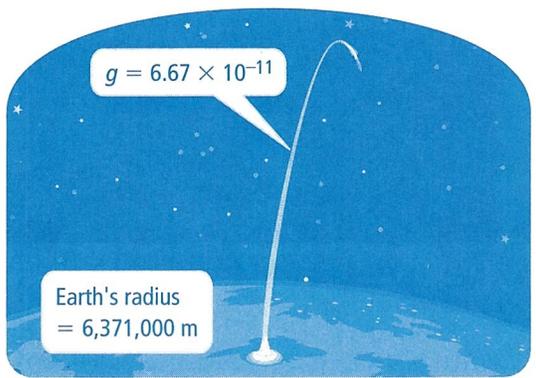
44. Complete the table to solve for the unknown value in the equation  $y = \sqrt[3]{2x + z} - 12$ , using the given values in each row.

y	x	z
0	462	
-3		439
-10	1.25	
3		16

45. **SAT/ACT** What is the solution to the equation  $(x^2 + 5x + 25)^{\frac{3}{2}} = 343$ ?
- (A) -8 only
  - (B) 3 only
  - (C) 77 only
  - (D) -8 and 3
  - (E) There are no solutions.

46. **Performance Task** Escape velocity is the velocity at which an object must be traveling to leave a star or planet without falling back to its surface or into orbit. Escape velocity,  $v$ , depends on the gravitational constant,  $G$ , the mass,  $M$ , and radius,  $r$ , of the star or planet.

$$v = \sqrt{\frac{2GM}{r}}$$



**Part A** Rewrite the equation to solve for mass.

**Part B** The escape velocity of Earth is 11,200 m/s and its radius is 6,371,000 m. The gravitational constant is  $6.67 \times 10^{-11}$ . What is Earth's mass in kilograms?