EXPLORE & REASON
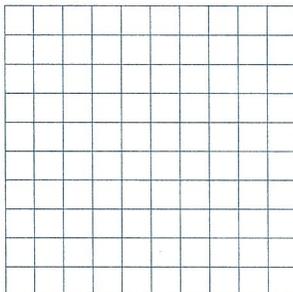
Consider the formula for the area of a square: $A = s^2$.

A. Graph the function that represents area as a function of side length.

B. On the same set of axes, graph the function that represents side length as a function of area.

C. **Look for Relationships** How are the two graphs related?
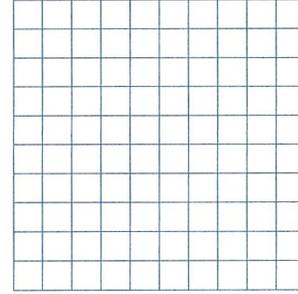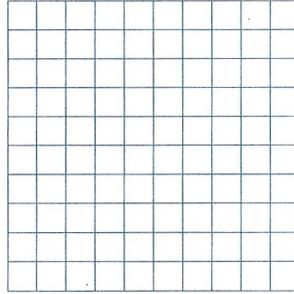
HABITS OF MIND

Communicate Precisely What is the domain and range of each function?

EXAMPLE 1 ☑ **Try It!** **Graph Square Root and Cube Root Functions**
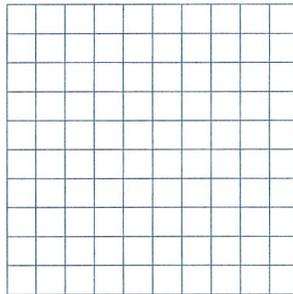
1. Graph the following functions. What are the domain and range of each function? Is the function increasing or decreasing?

    a. $f(x) = \sqrt{x - 5}$

    b. $g(x) = \sqrt[3]{x + 1}$

EXAMPLE 2 ☑ **Try It!** **Graph a Transformation of a Radical Function**

2. Graph $g(x) = \frac{1}{2}\sqrt{x - 1} - 3$. What transformations of the graph of $f(x) = \sqrt{x}$ produce the graph of g? What is the effect of the transformations on the domain and range of g?

HABITS OF MIND

Use Structure How does the graph of $y = \sqrt{x - a} + b$ compare to the graph of $y = \sqrt{x}$?

EXAMPLE 3 ☑ **Try It!** **Rewrite Radical Functions to Identify Transformations**
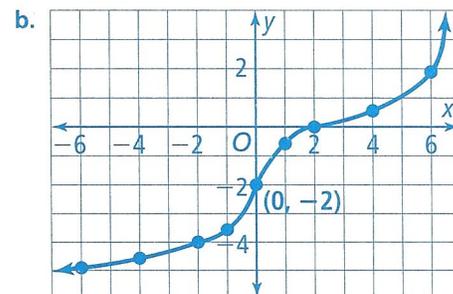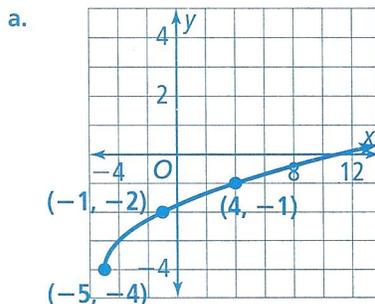
3. What transformations of the parent graph of $f(x) = \sqrt{x}$ produce the graphs of the following functions?

  a. $m(x) = \sqrt{7x - 3.5} - 10$

  b. $j(x) = -2\sqrt{12x} + 4$

EXAMPLE 4 ☑ **Try It!** **Write an Equation of a Transformation**

4. What radical function is represented in each graph below?

  a.

  b.

HABITS OF MIND

Model With Mathematics What is an example of a radical function whose domain is $x \geq -3$ and range is $y \geq 2$?

EXAMPLE 5 ☑ **Try It!** **Interpret a Radical Function Model**

5. Use the same function as in Example 5. Suppose Sasha's brother walks through elevations ranging from 8 ft to 48 ft. What are the minimum and maximum distances that he can see?

HABITS OF MIND

Generalize What transformations result in a cube root function being an odd function?

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you use what you know about transformations of functions to graph radical functions?
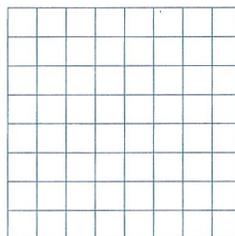
2. Error Analysis Parker said the graph of the radical function $g(x) = -\sqrt{x+2} -1$ is a translation 2 units left and 1 unit down from the parent function $f(x) = \sqrt{x}$. Describe and correct the error.

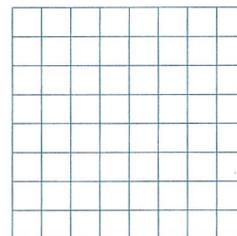**3. Reason** What effect does a have on the graph of $f(x) = a\sqrt{x}$?

Do You KNOW HOW?
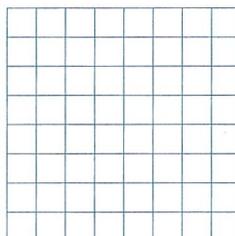
Graph each function. Then identify its domain and range.
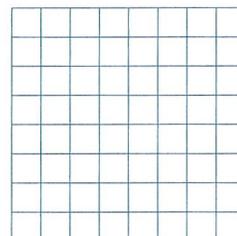
4. $f(x) = \sqrt{x-2}$

5. $f(x) = \sqrt[3]{x+2}$

6. $f(x) = \sqrt{x+1} - 2$

7. $f(x) = \sqrt[3]{x-3} + 2$

8. $f(x) = 3\sqrt{x-5}$
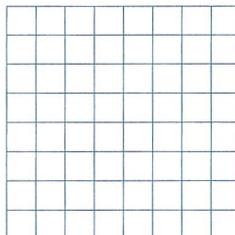
9. $f(x) = \frac{1}{2}\sqrt[3]{x} + 1$
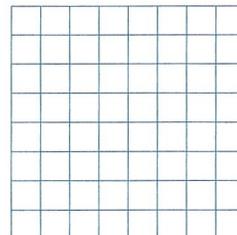
10. The volume of a cube is a function of the cube's side length. The function can be written as $V(s) = s^3$, where s is the side length and V is the volume.

   a. Express a cube's side length as a function of its volume, $s(V)$.

   b. Graph $V(s)$ and $s(V)$. What are the domain and range of the functions? Explain.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

11. **Communicate Precisely** What is the domain and range of the radical function $h(x) = \sqrt{x + a} + b$? Is the function increasing or decreasing? Explain.

12. **Model with Mathematics** The graph of a cube root function has a horizontal translation that is three times the vertical translation. The vertical translation is negative.

    a. Write a function, g, that has these attributes.

    b. Graph your function and the parent function, f, to verify it is correct.

13. **Error Analysis** Helena is trying to write a radical function that is represented by the graph below. Describe and correct the error Helena made in writing the radical function.
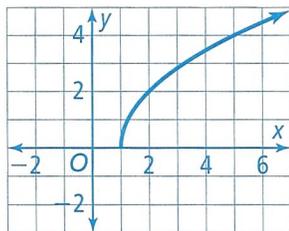
$f(x) = \sqrt{x - 1}$ ✗

14. **Higher Order Thinking** Rewrite the radical function $g(x) = \sqrt[3]{8x + 64} - 3$ to identify the transformations from the parent graph of $f(x) = \sqrt[3]{x}$. Explain how you rewrote the radical function.

15. **Reason** The parent function $f(x) = \sqrt{x}$ and a transformation of the parent function, $g(x)$, are reflections of each other over the x-axis. Write the function $g(x)$.

16. **Mathematical Connections** How do the transformations of a radical function compare to the transformations of an absolute value function?

PRACTICE & PROBLEM SOLVING

PRACTICE

Graph the following functions. State the domain and range. Is the function increasing or decreasing? SEE EXAMPLE 1

17. $f(x) = \sqrt{x} + 2$

18. $f(x) = \sqrt[3]{x} - 4$

19. $f(x) = \sqrt[3]{x - 8}$

20. $f(x) = \sqrt{x + 6}$

21. Graph $f(x) = \sqrt[3]{x}$ and $g(x) = 3\sqrt[3]{x + 9} - 8$. What transformations of the graph of f produce the graph of g? What effect do the transformations have on the domain and range of g? SEE EXAMPLE 2

Rewrite the following radical functions to identify their transformations from the parent graph
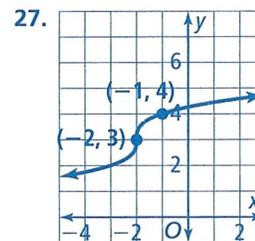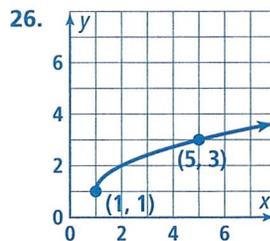$f(x) = \sqrt{x}$. SEE EXAMPLE 3

22. $f(x) = \sqrt{16x}$

23. $f(x) = \sqrt{25x + 75}$

24. $f(x) = \sqrt{9x - 45}$

25. $f(x) = \sqrt{4x - 24} - 6$

What radical function is represented in each graph? SEE EXAMPLE 4

26.

27.

28. The hull speed, y, measured in knots, of a sailboat can be estimated by the function $y = 1.34\sqrt{x}$, where x is the waterline length of the sailboat, in feet. Luis works at a sailboat rental business with boats that have a waterline length between 25 ft and 64 ft. SEE EXAMPLE 5

a. Graph the relationship between the hull speed of a sailboat and its waterline length.

b. What are the minimum and maximum hull speeds of the sailboats at the rental business?

PRACTICE & PROBLEM SOLVING

APPLY

29. **Make Sense and Persevere** The radius of a sphere can be found using the function $r = \sqrt[3]{\dfrac{3V}{4\pi}}$, where V is the volume of the sphere. Heather filled a basketball with 448.92 in.^{3} of air.

$V = 448.92$ in.^{3}
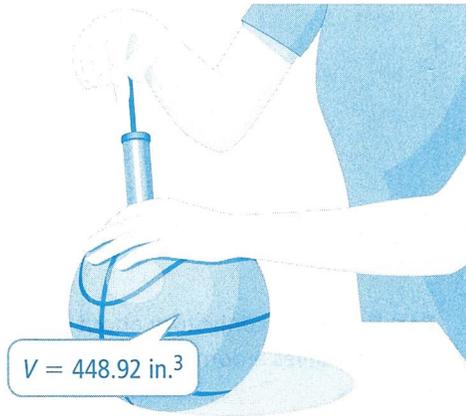
a. Graph the function.

b. Identify the domain and range of the graph.

c. Do the domain and range make sense in this context? Explain.

d. What is the length of the radius of the basketball?

30. A formula for calculating the distance to the horizon is $d = \sqrt{\dfrac{h}{0.57}}$, where d is the distance to the horizon, in miles, and h is the height above the surface, in feet.

Horizon 5 mi

a. Graph the function.

b. **Reason** What is your height above the surface if you can see a distance of 5 mi to the horizon?

☑ ASSESSMENT PRACTICE

31. Choose yes or no to tell whether the function is an odd function.

a. $f(x) = 5\sqrt{x - 10} - 12$ ○ Yes ○ No

b. $f(x) = \frac{1}{4}\sqrt[3]{x}$ ○ Yes ○ No

c. $f(x) = \frac{1}{2}\sqrt{x + 8} - 1$ ○ Yes ○ No

d. $f(x) = 6\sqrt[3]{x}$ ○ Yes ○ No

e. $f(x) = 9\sqrt[3]{x - 7} + 8$ ○ Yes ○ No

Part A Identify the domain and range of the function $f(x) = \sqrt[n]{x}$ when $n = 4, 5, 6, 7,$ and 8.

32. SAT/ACT Which function has a graph with domain $x \geq -1$ and range $y \geq -2$?

Ⓐ $f(x) = \sqrt{x - 1} + 2$

Ⓑ $f(x) = \sqrt[3]{x + 1} - 2$

Ⓒ $f(x) = \sqrt[3]{x - 1} + 2$

Ⓓ $f(x) = \sqrt{x + 1} - 2$

Part B Make a conjecture about the values of n that gives a domain and range of all real numbers.

33. Performance Task The table shows the domain and range of the function $f(x) = \sqrt[n]{x}$ for different values of n, where x is a positive real number.

Part C Make a conjecture about the values of n that gives a domain of $x \geq 0$ and a range of $y \geq 0$.

n	Domain of $f(x) = \sqrt[n]{x}$	Range of $f(x) = \sqrt[n]{x}$
1	All real numbers	All real numbers
2	$x \geq 0$	$y \geq 0$
3	All real numbers	All real numbers
4		
5		
6		
7		
8		