

CRITIQUE & EXPLAIN

Olivia was practicing evaluating and simplifying expressions. Her work for three expressions is shown.

- $24^2 = 400 + 16 = 416$
- $3^6 = 9(27) = 270 - 27 = 243$
- $\sqrt{625} = \sqrt{400} + \sqrt{225} = 20 + 15 = 35$

A. Is Olivia's work in the first example correct? Explain your thinking.

B. Is Olivia's work in the second example correct? Explain your thinking.

C. Is Olivia's work in the third example correct? Explain your thinking.

D. **Make Sense and Persevere** What advice would you give Olivia on simplifying expressions?

HABITS OF MIND

Construct Arguments You know that $3^2 + 4^2 = 5^2$. Does $\sqrt{3^2} + \sqrt{4^2} = \sqrt{5^2}$? If not, how could you rewrite the equation using radicals so that it is true?

**EXAMPLE 1** **Try It! Use Properties of Exponents**

1. How can you rewrite each expression using the properties of exponents?

a. $\left(\frac{3}{32^{\frac{2}{5}}}\right)^{\frac{1}{2}}$

b. $2a^{\frac{1}{3}}(ab^{\frac{1}{2}})^{\frac{2}{3}}$

EXAMPLE 2 **Try It! Use Properties of Exponents to Rewrite Radicals**

2. How can you rewrite each expression?

a. $\sqrt[4]{81a^8b^5}$

b. $\sqrt[3]{\frac{x^4y^2}{125x}}$

HABITS OF MIND

Make Sense and Persevere What do you have to check to be sure that an expression is in simplest radical form?

EXAMPLE 3 **Try It! Rewrite the Product or Quotient of a Radical**

3. What is the reduced radical form of each expression?

a. $\sqrt[5]{\frac{7}{16x^3}}$

b. $\sqrt[4]{27x^2} \cdot \sqrt{3x}$



EXAMPLE 4  **Try It! Add and Subtract Radical Expressions**

4. How can you rewrite each expression in a simpler form?

a. $\sqrt[3]{2,000} + \sqrt{2} - \sqrt[3]{128}$

b. $\sqrt{20} - \sqrt{600} - \sqrt{125}$

HABITS OF MIND

Critique Reasoning Divit says that you can simplify the product of any two radical expressions, but not necessarily the sum. Is he correct? Give an example.

EXAMPLE 5  **Try It! Multiply Binomial Radical Expressions**

5. Multiply.

a. $(x - \sqrt{10})(x + \sqrt{10})$

b. $\sqrt{6}(5 + \sqrt{3})$

EXAMPLE 6  **Try It! Rationalize a Binomial Denominator**

6. What is the reduced radical form of each expression?

a. $\frac{5 - \sqrt{2}}{2 - \sqrt{3}}$

b. $\frac{-4x}{1 - \sqrt{x}}$

HABITS OF MIND

Reason Is the product of two irrational binomials always irrational? Explain.

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can properties of exponents and radicals be used to rewrite radical expressions?

2. **Vocabulary** How can you determine if a radical expression is in reduced form?

3. **Use Structure** Explain why $(-64)^{\frac{1}{3}}$ equals $-64^{\frac{1}{3}}$ but $(-64)^{\frac{1}{2}}$ does not equal $-64^{\frac{1}{2}}$.

4. **Error Analysis** Explain the error in Julie's work in rewriting the radical expression.

$$\sqrt{-3} \cdot \sqrt{-12} = \sqrt{-3(-12)} = \sqrt{36} = 6$$

Do You KNOW HOW?

What is the reduced radical form of each expression?

5. $49^{\frac{3}{4}} \cdot 49^{-\frac{1}{4}}$

6. $\left(\frac{a^2b^8}{a^{\frac{1}{3}}}\right)^{\frac{3}{4}}$

7. $\sqrt[4]{1,024x^9y^{12}}$

8. $\sqrt[3]{\frac{4}{9m^2}}$

9. $\sqrt{63} - \sqrt{700} - \sqrt{112}$

10. $\sqrt{5}(6 + \sqrt{2})$

11. $\frac{3}{\sqrt{6}}$

12. $\frac{\sqrt{7}}{\sqrt{5} + 3}$

 **PRACTICE & PROBLEM SOLVING**
UNDERSTAND

13. Model With Mathematics In the expression $PV^{\frac{4}{3}}$, P represents the pressure and V represents the volume of a sample of a gas. Evaluate the expression for $P = 7$ and $V = 8$.

14. Reason Describe the possible values of k such that $\sqrt{32} + \sqrt{k}$ can be rewritten as a single term.

15. Error Analysis Explain why the following work is incorrect. Find the correct answer.

$$\begin{aligned} 5\left(4 - 5^{\frac{1}{2}}\right) &= 5(4) - 5\left(5^{\frac{1}{2}}\right) \\ &= 20 - 25^{\frac{1}{2}} \\ &= 15 \end{aligned}$$


16. Communicate Precisely Discuss the advantages and disadvantages of first rewriting $\sqrt{27} + \sqrt{48} + \sqrt{147}$ in order to estimate its decimal value.

17. Higher Order Thinking Write $\sqrt{\frac{4}{5}}$ in two different ways, one where the numerator is simplified and another where the denominator is rationalized.

18. Construct Arguments Justify each step used in simplifying the expression below.

$$\begin{aligned} \left(\frac{a^2}{a^4}\right)^{\frac{1}{5}} &= \left(a^{2-\frac{3}{4}}\right)^{\frac{1}{5}} \\ &= \left(a^{\frac{5}{4}}\right)^{\frac{1}{5}} \\ &= a^{\frac{1}{4}} \\ &= \sqrt[4]{a} \end{aligned}$$

PRACTICE & PROBLEM SOLVING

PRACTICE

What is the reduced radical form of each expression? SEE EXAMPLE 1

$$19. (3x^{\frac{1}{2}})(4x^{\frac{2}{3}}) \quad 20. 2b^{\frac{1}{2}}(3b^{\frac{1}{2}}c^{\frac{1}{3}})^2$$

$$21. (x^{\frac{1}{2}} \cdot x^{\frac{5}{12}})^4 \div x^{\frac{2}{3}} \quad 22. \left(\frac{16c^{14}}{81d^{18}}\right)^{\frac{1}{2}}$$

What is the reduced radical form of each expression? SEE EXAMPLE 2

$$23. \sqrt[3]{250y^2z^4} \quad 24. \sqrt[4]{256v^7w^{12}}$$

$$25. \sqrt{\frac{48x^3}{3xy^2}} \quad 26. \sqrt{\frac{56x^5y^5}{7xy}}$$

$$27. \sqrt[3]{216m} \quad 28. \sqrt[3]{\frac{250f^7g^3}{2f^2g}}$$

What is the reduced radical form of each expression? SEE EXAMPLE 3

$$29. \sqrt{x^5y^5} \cdot 3\sqrt{2x^7y^6} \quad 30. \sqrt[3]{\frac{18n^2}{24n}}$$

$$31. \sqrt[3]{3x^2} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{9x^3} \quad 32. \sqrt{\frac{162a}{6a^3}}$$

$$33. \sqrt[5]{2pq^6} \cdot 2\sqrt{2p^3q} \quad 34. \sqrt[3]{\frac{x^2}{9y}}$$

$$35. \sqrt[3]{6} \cdot \sqrt[3]{16} \quad 36. \sqrt[4]{\frac{2}{5x}}$$

What is the reduced radical form of each expression? SEE EXAMPLE 4

$$37. 4\sqrt[3]{81} - 2\sqrt[3]{72} - \sqrt[3]{24}$$

$$38. 6\sqrt{45y^2} - 4\sqrt{20y^2}$$

$$39. 3\sqrt{12} - \sqrt{54} + 7\sqrt{75}$$

$$40. \sqrt{32h} + 4\sqrt{98h} - 3\sqrt{50h}$$

Multiply. SEE EXAMPLE 5

$$41. (3\sqrt{p} - \sqrt{5})(\sqrt{p} + 5\sqrt{5})$$

$$42. (4m - \sqrt{3})(4m - \sqrt{3})$$

$$43. (3\sqrt{2} + 8)(3\sqrt{2} - 8)$$

$$44. \sqrt[3]{3}(5\sqrt[3]{9} - 4)$$

What is the reduced radical form of each expression? SEE EXAMPLE 6

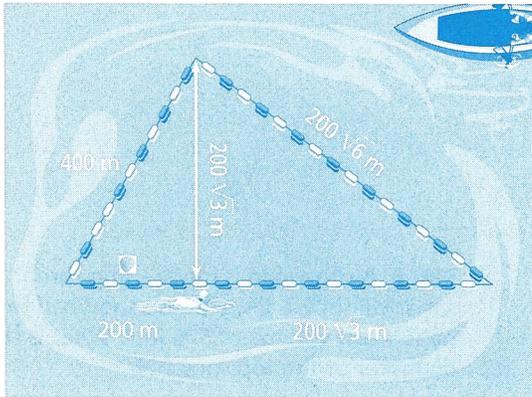
$$45. \frac{4}{1 - \sqrt{3}} \quad 46. \frac{20}{3 + \sqrt{2}}$$

$$47. \frac{3 + \sqrt{8}}{2 - 2\sqrt{8}} \quad 48. \frac{-2x}{3 + \sqrt{x}}$$

PRACTICE & PROBLEM SOLVING

APPLY

49. **Model With Mathematics** A triangular swimming area is marked off by a rope.

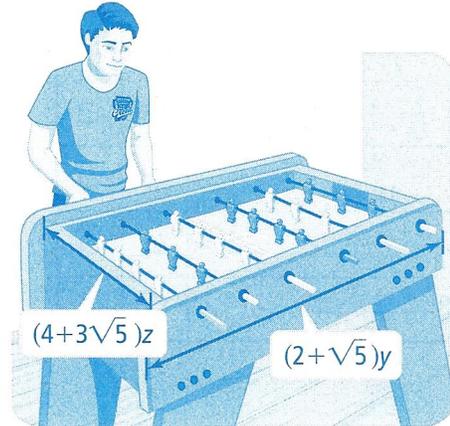


- a. If a woman swims around the perimeter of the swimming area, how far will she swim?

- b. What is the area of the roped off section?

50. **Use Structure** The interest rate r required to increase your investment p to the amount a in m months is found by $r = \left(\frac{a}{p}\right)^{\frac{1}{m}} - 1$. What interest rate would be required to increase your investment of \$3,600 to \$6,400 over 7 months? Round your answer to the nearest tenth of a percent.

51. **Use Structure** The length of a rectangle is $(2 + \sqrt{5})y$. The width is $(4 + 3\sqrt{5})z$. What is the area of the rectangle?



52. **Model With Mathematics** A rectangular boardroom table is $\sqrt{440}$ ft by $\sqrt{20}$ ft. Find its area.

ASSESSMENT PRACTICE

53. Aaron is rewriting $\frac{1+\sqrt{3}}{5-\sqrt{3}}$ into reduced radical form. Determine if Aaron would have written the steps below to show his work. Select Yes or No.

	Yes	No
$\frac{6 + 4\sqrt{3} - 3}{25 + 9}$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{5 + \sqrt{3} + 5\sqrt{3} + \sqrt{9}}{25 + 5\sqrt{3} - 5\sqrt{3} - \sqrt{9}}$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{4 + 3\sqrt{3}}{11}$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{8 + 6\sqrt{3}}{28}$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{5 + 6\sqrt{3} + 3}{25 - 3}$	<input type="checkbox"/>	<input type="checkbox"/>

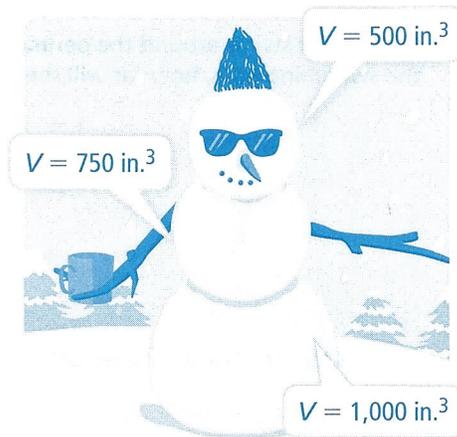
54. **SAT/ACT** Which expression cannot be rewritten as -10 ?

- (A) $\sqrt{25} \cdot \sqrt[3]{-8}$
- (B) $\sqrt[3]{-125} \cdot \sqrt[4]{16}$
- (C) $-\sqrt[3]{1,000}$
- (D) $-\sqrt{25} \cdot \sqrt[5]{-32}$
- (E) $\sqrt{4} \cdot -\sqrt[3]{125}$

55. **Performance Task** The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

Part A Use the formula to find r in terms of V . Rationalize the denominator.

Part B A snowman is made using three spherical snowballs. The top snowball for the head has a volume of 500 in.^3 . What is the diameter of the top snowball?



Part C The volumes of the other two snowballs are 750 in.^3 and $1,000 \text{ in.}^3$. How tall is the snowman?