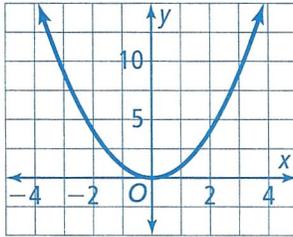


## EXPLORE & REASON

The graph shows  $y = x^2$ .



A. Find *all* possible values of  $x$  or  $y$  so that the point is on the graph.

(a)  $(2, \quad)$     (b)  $(3, \quad)$     (c)  $(-3, \quad)$     (d)  $(5, \quad)$

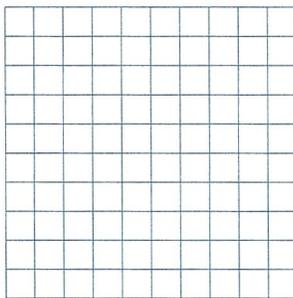
(e)  $(\quad, 4)$     (f)  $(\quad, -16)$     (g)  $(\quad, 7)$     (h)  $(\quad, 5)$

B. **Communicate Precisely** Write a precise set of instructions that show how to find an approximate value of  $\sqrt{13}$  using the graph.

C. Draw a graph of  $y = x^3$ . Use the graph to approximate each value.

(a)  $\sqrt[3]{5}$                       (b)  $\sqrt[3]{-5}$                       (c)  $\sqrt[3]{8}$

(d) A solution to  $x^3 = 5$     (e) A solution to  $x^3 = -5$     (f) A solution to  $x^3 = 8$



### HABITS OF MIND

**Look for Relationships** How is  $\sqrt[6]{5}$  related to  $\sqrt[3]{5}$ ?

# 5-1

## $n$ th Roots, Radicals, and Rational Exponents

**EXAMPLE 1**  **Try It! Find All Real  $n$ th Roots**

1. Find the specified roots of each number.
  - a. real fourth roots of 81
  - b. real cube roots of 64

**EXAMPLE 2**  **Try It! Understand Rational Exponents**

2. Explain what each fractional exponent means, then evaluate.
  - a.  $25^{\frac{1}{2}}$
  - b.  $32^{\frac{2}{5}}$

**HABITS OF MIND**

**Generalize** What is true about the denominators of fractional exponents in which absolute value must be considered?

**EXAMPLE 3**  **Try It! Evaluate Expressions With Rational Exponents**

3. What is the value of each expression? Round to the nearest hundredth if necessary.
  - a.  $-(16^{\frac{3}{4}})$
  - b.  $\sqrt[5]{3.5^4}$

**EXAMPLE 4** **Try It! Simplify  $n$ th Roots**

4. Simplify each expression.

a.  $\sqrt[3]{-8a^3b^9}$

b.  $\sqrt[4]{256x^{12}y^{24}}$

**HABITS OF MIND**

**Make Sense and Persevere** What is an example of a variable expression that has both a cube root and a fourth root which can be simplified to an expression without a radical?

**EXAMPLE 5** **Try It! Use  $n$ th Roots to Solve Equations**5. a. Solve the equation  $5x^3 = 320$ .      b. Solve the equation  $2p^4 = 162$ .**EXAMPLE 6** **Try It! Use  $n$ th Roots to Solve Problems**6. One cube has an edge length 3 cm shorter than the edge length of a second cube. The volume of the smaller cube is  $200 \text{ cm}^3$ . What is the volume of the larger cube?**HABITS OF MIND**

**Communicate Precisely** What are the steps necessary to solve the equation  $ax^n = b$ ?

## Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are exponents and radicals used to represent roots of real numbers?

2. **Error Analysis** Kaitlyn said  $\sqrt[3]{10} = 10^3$ . Explain Kaitlyn's error.

3. **Vocabulary** In the radical expression  $\sqrt[5]{125}$ , what is the index? What is the radicand?

4. **Use Structure** Why is  $75^{\frac{3}{5}}$  equal to  $(75^{\frac{1}{5}})^3$ ?

5. **Construct Arguments** Anastasia said that  $(x^8)^{\frac{1}{4}} = \frac{x^8}{x^4} = x^4$ . Is Anastasia correct? Explain.

6. **Make Sense and Persevere** Is it possible for a rational exponent to be an improper fraction? Explain how  $27^{\frac{4}{3}}$  is evaluated or why it cannot be evaluated.

## Do You KNOW HOW?

Write each expression in radical form.

7.  $a^{\frac{1}{5}}$

8.  $7^{\frac{2}{3}}$

Write each expression in exponential form.

9.  $\sqrt[3]{b}$

10.  $\sqrt[4]{p^7}$

11. How many real third roots does 1,728 have?

12. How many real sixth roots does 15,625 have?

13. Solve the equation  $4x^3 = 324$ .

14. Solve the equation  $2x^4 = 2,500$ .

Simplify each expression.

15.  $\sqrt[3]{27x^{12}y^6}$

16.  $\sqrt[5]{-32x^5y^{30}}$

17. A snow globe is packaged in a cubic container that has volume  $64 \text{ in.}^3$ . A large shipping container is also a cube, and its edge length is 8 inches longer than the edge length of the snow globe container. How many snow globes can fit into the larger shipping container?

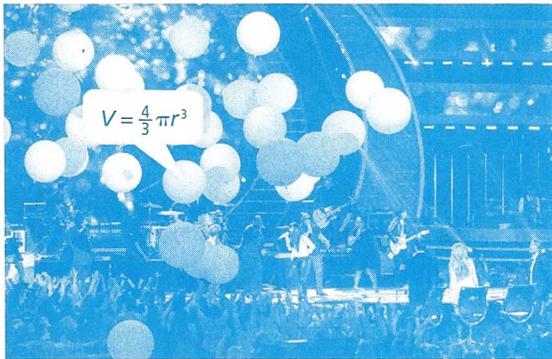


## PRACTICE & PROBLEM SOLVING

### UNDERSTAND

**18. Construct Arguments** Justice found that the fifth root of  $243x^{15}y^5$  is  $3x^3y$ . Is Justice correct? Explain your reasoning.

**19. Make Sense and Persevere** For a show, each sphere was inflated to have a volume of  $4,186\frac{2}{3}$  in.<sup>3</sup> Explain how to find the radius  $r$  of one of the inflated spheres. Use technology to compute your answer.



**20. Error Analysis** Describe and correct the error a student made in writing this exponential expression in radical form.

$$\begin{aligned} x^{\frac{4}{3}} &= (x^4)^{\frac{1}{3}} \\ (x^4)^{\frac{1}{3}} &= \sqrt[4]{x^3} \end{aligned}$$

**X**

**21. Construct Arguments** Determine whether  $\sqrt[3]{x^2}$  is equal to  $(\sqrt[3]{x})^2$ . Explain your reasoning.

**22. Use Structure** How many third roots does  $-512$  have? Explain your reasoning.

**23. Higher Order Thinking** The annual interest formula below calculates the final balance of an account,  $F$ , given a starting balance,  $S$ , and an interest rate,  $r$ , after 10 years.

$$F = S(1 + r)^{10}$$

When solving for  $r$ , why can the negative root be ignored?

**24. Mathematical Connections** The lengths of the two legs of a right triangle are 4 and 8. What is the length of the hypotenuse, in simplest radical form?

## PRACTICE & PROBLEM SOLVING

### PRACTICE

Find the specified roots of each number.

SEE EXAMPLE 1

25. the real fourth roots of 81

26. the real third roots of 343

27. the real fifth roots of 1,024

28. the real square roots of 25

Rewrite each expression using a fractional exponent. SEE EXAMPLE 2

29.  $\sqrt[4]{16^2}$

30.  $\sqrt[6]{729}$

31.  $\sqrt[3]{x^2}$

32.  $\sqrt[4]{ab}$

What is the value of each expression? Round to the nearest hundredth if necessary. SEE EXAMPLE 3

33.  $\sqrt[4]{25^2}$

34.  $-\sqrt[3]{125^5}$

Simplify each expression. SEE EXAMPLE 4

35.  $\sqrt[3]{8y^9}$

36.  $\sqrt[4]{q^{12}z^4}$

37.  $\sqrt[6]{729a^{24}b^{18}}$

38.  $\sqrt[8]{v^8g^{40}}$

Solve each equation. SEE EXAMPLE 5

39.  $1,125 = 9x^3$

40.  $6,480 = 5w^4$

41.  $270 = 10q^3$

42.  $256 = 4h^6$

43. A small cube has the volume shown. Its side length is 1.5 in. less than a second, larger cube. What is the volume of the larger cube?

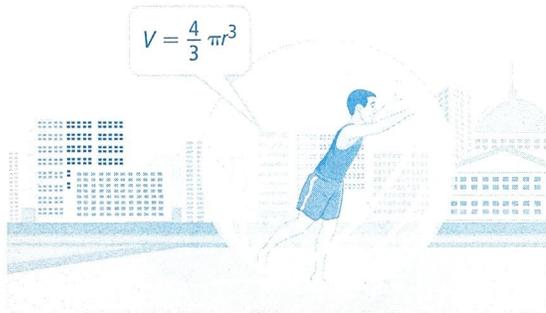
SEE EXAMPLE 6



**PRACTICE & PROBLEM SOLVING**

**APPLY**

44. **Model With Mathematics** A water-walking ball has a volume of approximately  $4.19 \text{ m}^3$ . What is the radius,  $r$ , of the ball?



45. **Make Sense and Persevere** Ahmed received a box of gifts. The box is a rectangular prism with the same height and width, and the length is twice the width. The volume of the box is  $3,456 \text{ in.}^3$ . What is the height of the box?



46. **Make Sense and Persevere** Amelia's bank account earns interest annually. The equation shows her starting balance of \$200 and her balance at the end of four years, \$220.82. At what rate,  $r$ , did Amelia earn interest?

$$220.82 = 200(1 + r)^4$$

47. **Model With Mathematics** One measure of a patient's body surface area is found using the expression  $\sqrt{\frac{H \cdot W}{3,600}}$ . Write this with a fractional exponent.

**ASSESSMENT PRACTICE**

48. Determine if each expression is another way to write  $b^{\frac{3}{4}}$ . Select Yes or No.

	Yes	No
a. $\sqrt[4]{b^3}$	<input type="checkbox"/>	<input type="checkbox"/>
b. $(b^3)^{\frac{1}{4}}$	<input type="checkbox"/>	<input type="checkbox"/>
c. $b^{\frac{4}{3}}$	<input type="checkbox"/>	<input type="checkbox"/>
d. $\sqrt[3]{b^4}$	<input type="checkbox"/>	<input type="checkbox"/>
e. $\frac{b^3}{b^4}$	<input type="checkbox"/>	<input type="checkbox"/>

49. **SAT/ACT** Which of the following is equivalent to  $\sqrt[6]{4,096x^{18}y^{30}}$ ?

- (A)  $682.7x^{15}y^{24}$
- (B)  $4x^{1.6}y^{1.8}$
- (C)  $4,096x^3y^5$
- (D)  $4x^3y^5$
- (E)  $682.7x^3y^5$

50. **Performance Task** A milk processing company uses cylindrical-shaped containers. The height of the container is equal to the diameter of the base.



**Part A** The volume of one container is about  $169.65 \text{ ft}^3$ . How much material is needed to make the lateral surface of the shipping container?

**Part B** The cargo hold of a ship is 20 ft high. What is the largest number of these shipping containers that could be stacked on top of each other inside the cargo hold?