**11.8 Hero's and Brahmagupta's Formulas**

**Lesson Objective:** *After studying this section, you will be able to:*

- *Find the areas of figures by using Hero's formula (triangle area)*
- *Find the areas of figures by using Brahmagupta's formula (inscribed quadrilaterals)*

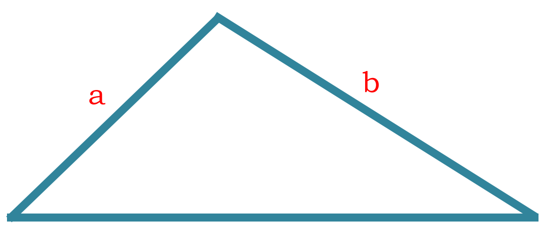
***<u>Area of Triangles:</u>***

The following ***triangle area*** formula was developed nearly 2000 years ago by a mathematician known as Hero of Alexandria!

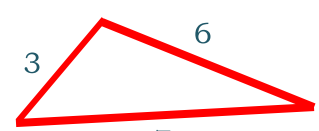
**Theorem 111: (Hero's Formula)** $A_{\Delta} = \sqrt{s(s-a)(s-b)(s-c)}$ ,

where a, b, and c are the lengths of the sides of the triangle

and s = semiperimeter = $\frac{a+b+c}{2}$




**Example:** Use ***Hero's formula*** to find the area of a triangle with sides 3, 6, and 7.




**Step 1:** Find the perimeter → 3 + 6 + 7 = 16
**Step 2:** Find the semiperimeter → 16 ÷ 2 = 8
**Step 3:** Replace variables in Hero's formula using corresponding values and evaluate!

**s** = 8, **a** = 3, **b** = 6, **c** = 7

$$A_{\Delta} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A_{\Delta} = \sqrt{8(8-3)(8-6)(8-7)}$$

$$A_{\Delta} = \sqrt{8(5)(2)(1)} = \sqrt{80} = \sqrt{(16)(5)} = \mathbf{4\sqrt{5}}$$

***<u>Suggestion:</u>*** Try Hero's formula to find the area of a familiar right triangle to see that it works! Check by using $A = \frac{bh}{2}$

Use sides lengths: 3, 4, and 5!

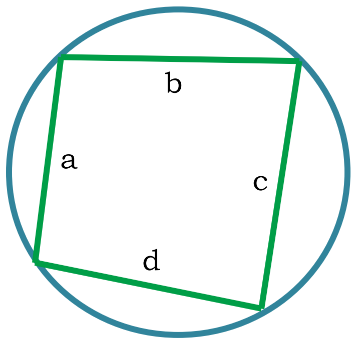
**Question:**

When should you use Hero's formula and when should you simply use $A_{\Delta} = \frac{bh}{2}$ ?

## ***Area of Cyclic Quadrilaterals***

A Hindu mathematician named **Brahmagupta** recorded a formula for deriving the area of an ***inscribed quadrilateral*** in about 628 A.D. Brahmagupta's formula can only be applied to quadrilaterals that are ***cyclic quadrilaterals***, meaning quadrilaterals that can be inscribed in circles.

**Theorem 112: (Brahmagupta's formula)**

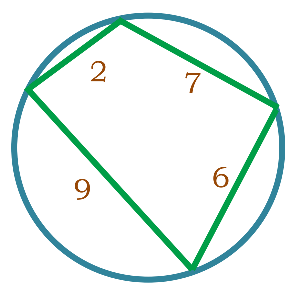



$$A_{cyclic\ quad} = \sqrt{(s-a)(s-b)(s-c)(s-d)},$$

where a, b, c, and d are the sides of the quadrilateral

and s = semiperimeter = $\frac{a+b+c+d}{2}$

**Example:** Use **Brahmagupta's formula** to find the area of a cyclic quadrilateral with sides lengths: 2 cm, 7 cm, 6 cm, and 9 cm.




**Step 1:** Find the perimeter → 2 + 7 + 6 + 9 = 24 cm

**Step 2:** Find the semiperimeter → (24 cm) ÷ 2 = 12 cm

**Step 3:** Substitute corresponding values into Brahmagupta's formula and evaluate.

s = 12, a = 2, b = 7, c = 6, d = 9

$$A_{cyclic\ quad} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$A_{cyclic\ quad} = \sqrt{(12-2)(12-7)(12-6)(12-9)}$$

$$A_{cyclic\ quad} = \sqrt{(10)(5)(6)(3)} = \sqrt{900}$$

$$A_{cyclic\ quad} = \sqrt{900} = \mathbf{30\ cm^2}$$