

Algebra 2 Topic 6: Exponential and Logarithmic Functions		Estimate Time Frame: 6 Block Days
Essential Standards: A.2, F.1. c, F.1.d, F.4, F.8.a, F.8.b, F.9.a, F.10, A.17		
Assessment Resource: enVision Topic 6 and Formative Assessment Lesson (FAL): <a href="#">Exponential FAL</a>		
FCPS Supporting Links		Additional Supporting Links
<a href="#">Pacing Guide</a> <a href="#">enVision Algebra 2 Standards Crosswalk Resource</a> <a href="#">FCPS P-12 Mathematics Guidance Document</a>		<a href="#">Kentucky Academic Standards KSA Blueprint</a> <a href="#">Achieve the Core Operations and Algebraic Thinking Progressions Target of the Standards</a> - conceptual, procedural & application <a href="#">Three-Reads Routine</a> <a href="#">Notice and Wonder Routine</a>  <a href="#">MILC Resources Topic 6 Exponential and Logarithmic Functions</a>  <b><i>enVision Algebra 2 Teacher Guide: page 282A to 282D for specific Topic 6 Focus-Coherence-Rigor</i></b>
Big Ideas		
Algebra and the coordinate plane can be used to solve problems involving geometric concepts. Congruent or supplementary angles formed by two lines and a transversal can be used to prove the lines are parallel.		
Essential Questions		Common Preconceptions/Misconceptions
<ul style="list-style-type: none"> <li>• How do you use exponential functions to model situations and solve problems?</li> <li>• When does a function best model a situation?</li> <li>• What kinds of transformations will affect the asymptote of an exponential function?</li> </ul>		<ul style="list-style-type: none"> <li>• Students may incorrectly confuse the rate of decay and the decay factor.</li> <li>• Some students may confuse the placement of the various numbers when converting to another form of the equation with logarithms.</li> <li>• Some students may think <math>e</math> is a variable and not a constant. Explain that <math>e</math> is an irrational number called Euler's number.</li> </ul>

<ul style="list-style-type: none"><li>•Why can't you divide an annual interest rate by 4 to obtain a quarterly interest rate?</li><li>•How do you use logarithmic functions to model situations and solve problems?</li></ul>		
Standards for Mathematical Practices	Kentucky Interdisciplinary Literacy Practices (KILP)	
<p><a href="#"><u>MP.1. Make sense of problems and persevere in solving them.</u></a></p> <p><a href="#"><u>MP.2. Reason abstractly and quantitatively.</u></a></p> <p><a href="#"><u>MP.3. Construct viable arguments and critique the reasoning of others.</u></a></p> <p><a href="#"><u>MP.4. Model with mathematics.</u></a></p> <p><a href="#"><u>MP.5. Use appropriate tools strategically.</u></a></p> <p><a href="#"><u>MP.6. Attend to precision.</u></a></p> <p><a href="#"><u>MP.7. Look for and make use of structure.</u></a></p> <p><a href="#"><u>MP.8. Look for and express regularity in repeated reasoning.</u></a></p> <p><i>enVision Teacher Guide: page 282D for specific Math Practice suggestions</i></p>	<ol style="list-style-type: none"><li>1. Recognize that text is anything that communicates a message.</li><li>2. Employ, develop, and refine schemas to understand and create text.</li><li>3. View literacy experiences as transactional, interdisciplinary, and transformational.</li><li>4. Utilize receptive and expressive language arts to better understand self, others, and the world.</li><li><b>5. Apply strategic practices, with scaffolding and then independently, to approach new literacy tasks.</b></li><li><b>6. Collaborate with others to create new meaning.</b></li><li><b>7. Utilize digital resources to learn and share with others.</b></li><li><b>8. Engage in specialized, discipline-specific literacy practices.</b></li><li><b>9. Apply high-level cognitive processes to think deeply and critically about text.</b></li><li>10. Develop a literacy identity that promotes lifelong learning.</li></ol> <p><i>Incorporating texts into math instruction fosters interdisciplinary learning for a more engaging educational experience.</i></p>	
Essential Standards	Sample Learning Intentions & Success Criteria	HQIR/Resource Considerations
Cluster: Understand the concept of a function and use function notation.		
<p><b>KY.HS.F.1</b> Understand properties and key features of functions and the different ways functions can be represented.</p>	<p>We are learning to graph and interpret key features of graphs of exponential and logarithmic functions.</p> <ul style="list-style-type: none"><li>• I can identify the domain and range of an exponential and logarithmic function.</li><li>• I can sketch a function graph.</li></ul>	<ul style="list-style-type: none"><li>• Topic 6-1</li></ul> <p>A function is often described and understood in terms of the output behavior, or over what input values it is</p>

<p><b>c.</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.</p> <p><b>d.</b> Graph exponential and logarithmic functions, showing intercepts and end behavior.</p> <p><b>e.</b> (+) Graph trigonometric functions, showing period, midline, and amplitude.</p> <p><b>f.</b> (+) Graph piecewise functions, including step functions.</p> <p><b>g.</b> (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior</p> <p><b>MP.4, MP.5, MP.8, KILP.1, KILP.7</b></p> <p><i>Supporting Standard(s): KY.HS.F.9, KY.HS.F.12, KY.HS.F.13, KY.HS.F.14</i></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>We are learning to interpret key features of graphs and tables of functions that model relationships between two quantities and sketch the graph.</p> <ul style="list-style-type: none"> <li>• I can sketch graphs that accurately represent the key features described verbally, including shape, direction, and position of essential points.</li> <li>• I can interpret key features of graphs and tables given a verbal description.</li> </ul>	<p>increasing, decreasing, or constant.</p> <p>Important questions include, “For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large in magnitude?”</p>
<p><b>Cluster: Analyze functions using different representations.</b></p>		
<p><b>KY.HS.F.4</b> Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★</p> <p><b>b.</b> Graph square root, cube root, and absolute value functions.</p> <p><b>d.</b> Graph exponential and logarithmic functions, showing intercepts and end behavior.</p>	<p>I am learning to graph exponential functions, demonstrating an understanding of intercepts and end behavior.</p> <ul style="list-style-type: none"> <li>• I can identify the end behavior and key features of polynomial functions.</li> <li>• I can apply their understanding of exponential functions to solve problems in various contexts, such as population growth, compound interest, and</li> </ul>	<ul style="list-style-type: none"> <li>• Topic 6-1</li> <li>• Topic 6-4</li> </ul>

<p><b>e.</b> (+) Graph trigonometric functions, showing period, midline, and amplitude.</p> <p><b>f.</b> (+) Graph piecewise functions, including step functions.</p> <p><b>g.</b> (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior</p> <p><b>MP.4, MP.5, MP.8, KILP.1, KILP.7</b></p> <p><i>Supporting Standard(s): KY.HS.F.9, KY.HS.F.12, KY.HS.F.13, KY.HS.F.14</i></p> <p><input type="checkbox"/> Conceptual   <input type="checkbox"/> Procedural   <input type="checkbox"/> Application</p>	<p>radioactive decay.</p>	
<p><b>Cluster: Build new functions from existing functions.</b></p>		
<p><b>KY.HS.F.10</b> Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents using technology.</p> <p><b>MP.1, MP.7, KILP.1, KILP.5, KILP.7</b></p> <p><i>Supporting Standard(s): KY.HS.F.8, KY.HS.F.9, KY.HS.A.12, KY.HS.A.16</i></p> <p><input type="checkbox"/> Conceptual   <input type="checkbox"/> Procedural   <input type="checkbox"/> Application</p>	<p>We are learning to use the inverses of logarithmic and exponential equations to solve those equations.</p> <ul style="list-style-type: none"> <li>• I can find equations of the inverses of exponential and logarithmic functions.</li> <li>• I can evaluate and simplify logarithms.</li> <li>• I can use logarithms to solve exponential models.</li> </ul>	<ul style="list-style-type: none"> <li>• Topic 6-3</li> <li>• Topic 6-4</li> </ul> <p>Formative Assessment Lesson (FAL) <a href="#">Exponential FAL</a></p> <p>Students can use the inverses of simple logarithmic and exponential equations to solve those equations.</p> <p>The inverse relationship between logarithmic and exponential functions is</p>

		special because each function's inverse is also a function.
<b>Cluster: Build new functions from existing functions.</b>		
<p><b>KY.HS.F.8</b> Understand the effects of transformations on the graph of an exponential and logarithmic function. <b>MP.3, MP.5</b></p> <p>a. Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); <b>and</b> find the value of <math>k</math>, given the graphs.</p> <p>b. Experiment with cases and illustrate an explanation of the effects on the graph using technology.</p>	<p>We are learning to understand the effects of transformation on graphs.</p> <ul style="list-style-type: none"> <li>I can identify the value of <math>k</math>, given a graph.</li> <li>I can apply their understanding of functions using technology.</li> </ul>	<ul style="list-style-type: none"> <li>Topic 6-1</li> <li>Topic 6-4</li> </ul> <p>Mastery of this standard includes recognizing even and odd functions from their graphs and algebraic expressions.</p>
<b>Cluster: Build new functions from existing functions.</b>		
<p><b>KY.HS.F.9</b> Find inverse functions.</p> <p>a. Given the equation of an invertible function, find the inverse.</p> <p>b. (+) Verify by composition that one function is the inverse of another.</p> <p>c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>d. (+) Produce an invertible function from a non-invertible function by restricting the domain.</p> <p><b>MP.2, MP.6</b></p>	<p>We are learning to find the inverse functions when provided with the equation of an invertible function.</p> <ul style="list-style-type: none"> <li>I can identify an invertible function and understand its unique inverse function.</li> <li>I can apply algebra to find the inverse of a given invertible function, such as solving for the variable in terms of the output or interchanging the roles of input and output variables.</li> <li>I can verify the correctness of the inverse function by composing it with the original function and observing that the result is the identity function.</li> <li>I can graphically represent the inverse</li> </ul>	<ul style="list-style-type: none"> <li>Topic 6-3</li> <li>Topic 6-4</li> </ul> <p>a. Students can complete the process of finding the inverse when given an equation for an invertible function.</p> <p>b-d. Students need a formal sense of inverse functions.</p> <p>Students understand that a function and its inverse</p>

<input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application	function by reflecting the original function's graph across the line $y = x$ . <ul style="list-style-type: none"> <li>I can apply the inverse function to solve equations and real-world problems involving the original function.</li> </ul>	describe the exact same relationship, but in different ways.
<b>Cluster: Interpret the structure of expressions.</b>		
<p><b>KY.HS.A.2</b> Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms.</p> <p><b>MP.7, MP.8, KILP.2, KILP.6</b></p> <p><i>Supporting Standard(s): KY.HS.A.3, KY.HS.A.7</i></p> <input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application	We are learning to rewrite expressions in equivalent forms. <ul style="list-style-type: none"> <li>I can identify ways to factor expressions based on the structure of the expression.</li> <li>I can apply algebraic properties and rules to rewrite expressions, demonstrating proficiency in simplifying and manipulating expressions to achieve equivalent forms.</li> </ul>	<ul style="list-style-type: none"> <li>Topic 6-2</li> <li>Topic 6-5</li> <li>Topic 6-6</li> </ul> <p><b>3-Act Math Task</b> - The Crazy Conditioning (Use after Lesson 6-2)</p>
<b>Cluster: Understand solving equations as a process of reasoning and explain the reasoning.</b>		
<p><b>KY.HS.A.17</b> Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise.</p> <p><b>MP.3, MP.5, MP.7, KILP.1, KILP.2, KILP.6</b></p> <p><i>Supporting Standard(s): KY.HS.A.15</i></p> <input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application	We are learning to solve exponential and logarithmic equations with one variable. <ul style="list-style-type: none"> <li>I can use logarithms to express the solutions to exponential models.</li> <li>I can substitute solutions back into the original equation to verify that they satisfy it.</li> </ul>	<ul style="list-style-type: none"> <li>Topic 6-6</li> </ul>
<b>Attending to the Standards for Mathematical Practice</b>		
Students use technology to explore how changing the value of $k$ impacts the function's graph (MP.5).		

Students use graphical representations to create plausible arguments about the effects of transformations instead of relying on computational rules (MP.3).

### Supporting Standards

**KY.HS.F.5.b.** Use the properties of exponents to interpret expressions for exponential functions and classify the exponential function as representing growth or decay. **MP.3, MP.6 Topic (6-2)**

**KY.HS.F.9** Find inverse functions.

a. Given the equation of an invertible function, find the inverse.

b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2, MP.6**

**KY.HS.F.12** Construct **exponential functions**, including **geometric sequences**, given a graph, given a description of a relationship, or two input-output pairs (including reading these from a table). **MP.7, MP.8 Topic (6-7)**

**KY.HS.F.13** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. **MP.7, MP.8**

**KY.HS.F.14** Interpret the parameters in an exponential function in terms of a context. **MP.1, MP.2 Topic (6-2)**

**KY.HS.F.16 (+)** Understand and use the unit circle. a. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. b. Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$  and use the unit circle to express the values of sine, cosine and tangent for  $\pi - x$ ,  $\pi + x$  and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number. c. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **MP.7, MP.8**

### Vocabulary

Exponential equation

Logarithmic equation

Decay factor

Exponential decay function

Exponential growth function

Growth factor

Compound Interest Natural base e logarithm
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\*Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

\*\* Mathematical Practices (A.MP. 1- 8) should be evidenced at some point throughout each unit, depending on the explored tasks. It is important to note that MP. 2 should support learning in every lesson.

\*\*\* Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather with other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.