Unit 4 Title	**Estimated Time Frame**
Right Triangles, Trigonometry, and Circles	**40 days or 20 block days**

Big Idea (s)

The properties of circles can be used to solve problems involving polygons, lines, and angles.
The angle measures and side lengths within a right triangle have many special relationships (Pythagorean Theorem, trigonometric ratios).
Dilations and similarity allow for applying trigonometric ratios in solving various real-world problems.

Essential Question(s)

How can algebra be used to express geometric properties?
What relationships exist among the sides and angles of a right triangle?
How can right triangles be used to model and solve real-world problems?
How are the Pythagorean Theorem and trigonometry useful?
How do trigonometric ratios relate angle measures to side lengths of right triangles?

How is the equation of a circle determined in the coordinate plane?
How is a tangent line related to the radius of a circle at the point of tangency?
How are chords related to their central angles and intercepted arcs?
How is the measure of an inscribed angle related to its intercepted arc?
How are the measures of angles, arcs, and segments formed by intersecting secant lines related?

Standards for Mathematical Practice (MP.) - The practice standards in bold describe expertise to be intentionally developed in this unit.	**Kentucky Interdisciplinary Literacy Practices (KILP.) -** The practice standards in bold describe expertise to be intentionally developed in Mathematics.
MP.1. Make sense of problems and persevere in solving them. **MP.2. Reason abstractly and quantitatively.** **MP.3. Construct viable arguments and critique the reasoning of others.** **MP.4. Model with mathematics.** **MP.5. Use appropriate tools strategically.** MP.6. Attend to precision. **MP.7. Look for and make use of structure.** **MP.8. Look for and express regularity in repeated reasoning.**	KILP.1 Recognize that text is anything that communicates a message. KLIP.2 Employ, develop, and refine schema to understand and create text. KILP.3 View literacy experiences as transactional, interdisciplinary and transformational. KILP.4 Utilize receptive & expressive language arts to better understand self, others, and the world. KILP.5 Apply strategic practices, with scaffolding & then independently, to approach new literacy tasks. **KILP.6 Collaborate with others to create new meaning.** **KILP.7 Utilize digital resources to learn and share with others.** **KILP.8 Engage in specialized, discipline specific literacy practices.** **KILP.9 Apply high level cognitive processes to think deeply and critically about text.** KILP.10.Develop a literacy identity that promotes lifelong learning.

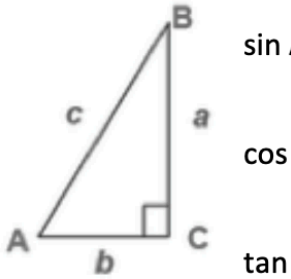
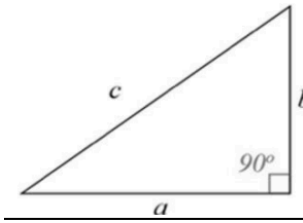
Common Preconceptions/Misconceptions

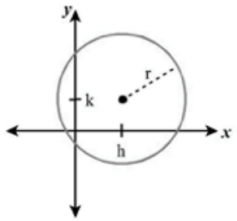
Students should have seen the formulas in these standards in the middle grades. What is different for high school students in Advanced Geometry is that these students are asked to justify *with formal and informal proofs.*

Students should explore with models.

Students should use technology to identify the result of cutting a three-dimensional object and the result of rotating two-dimensional objects.

As students analyze two-dimensional and three-dimensional shapes, they gain insights into the structure of specific shapes. For instance, students consider the two-dimensional figures resulting from removing the top of a shoe box or slicing an orange. Students compare and contrast the two-dimensional cross-sections of oranges when sliced at different locations or angles versus slicing. For an extension, students can compare conjectures from circles when slicing a cone at different locations or angles.

KAS Standards	Considerations	Samples of Learning Intentions and Success Criteria
Cluster: Define trigonometric ratios and solve problems involving right triangles.		
KY.HS.G.12 Understand the properties of right triangles. a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles (sine, cosine, and tangent). ☐ Conceptual ❑ Procedural ❑ Application b. Explain and use the relationship between the sine and cosine of complementary angles. ☐ Conceptual ☐ Procedural ❑ Application c. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★ ☐ Conceptual ☐ Procedural ☐ Application	 $\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$ $\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$ $\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$  $c^2 = a^2 + b^2$	We are learning to apply triangle theorems to solve problems involving right angles. ● I can use the Pythagorean Theorem to find missing sides in right triangles. ● I can use the Converse of the Pythagorean Theorem to classify a triangle as acute, right, or obtuse. ● I can find missing sides in 30°-60°-90° special right triangles. ● I can find missing sides in 45°-45°-90° special right triangles. We are learning to explain and use trigonometry relationships. ● I can explain sine and cosine relationships. ● I can use sine equals cosine of complementary angles to solve problems. We are learning to apply trigonometric ratios to solve problems involving right triangles.

MP.3, MP.4, KILP.6, KILP.7, KILP.9 *Supporting Standards:* KY.HS.G.13 (+) & KY.HS.G.14 (+), KY.HS.G.29		• I can use trigonometry to find missing sides in right triangles. • I can use trigonometry to find missing angles in right triangles. • I can use SOH-CAH-TOA to find the sine, cosine, and tangent of an acute angle of a triangle.
Cluster: Translate between the geometric description and the equation for a conic section.		
KY.HS.G.19 Understand the relationship between the algebraic form and the geometric representation of a circle. a. Write the equation of a circle of a given center and radius using the Pythagorean Theorem. b. **(+)** Derive and write the equation of a circle of a given center and radius using the Pythagorean Theorem. c. **(+)** Complete the square to find the center and radius of a circle given by an equation. ☐ Conceptual ☐ Procedural ❑ Application **MP.6, MP.8, KILP.1, KILP.2, KILP.7** *Supporting Standards:* KY.HS.G.1, KY.HS.G.15, KY.HS.G.21	 **$(x - h)^2 + (y - k)^2 = r^2$**	We are learning about the relationship between the algebraic form and the geometric representation of a circle. • I can define a circle. • I can use Pythagorean Theorem to find the radius of a circle. • I can use the radius and center to find the equation of a circle. • I can write an equation of a circle from the center and radius.
Cluster: Understand and apply theorems about circles		
KY.HS.G.16 Identify and describe relationships among angles and segments within the context of circles involving: a. Recognize differences between and properties of inscribed, central, and circumscribed angles.	Students recognize and apply relationships, including the relationship between central, inscribed, and circumscribed angles, inscribed angles on a diameter are right angles, and the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	We are learning to identify and describe relationships among angles and segments within the context of circles. • I can identify inscribed angles, radii, chords, central angles, circumscribed angles, diameter, and tangent. We are learning to understand relationships

b. Understand relationships between inscribed angles and the diameter of a circle. c. Understand the relationship between the radius of a circle and the line drawn through the point of tangency on that radius. **MP.3, MP.5, and MP.7, KILP.2, KILP.5, KILP.8** *Supporting Standards: KY.HS.G.1, KY.HS.G.17 (+) & KY.HS.G.18 (+)* ☐ Conceptual ☐ Procedural ❑ Application		between inscribed angles. ● I can recognize that inscribed angles on a diameter are right angles. ● I can recognize that the radius of a circle is perpendicular to the radius at the point of tangency. ● I can examine the relationship between central, inscribed, and circumscribed angles by applying theorems about their measures.

Supporting Standards

KY.HS.G.1 Know and apply precise definitions of the language of Geometry: a. Understand properties of line segments, angles and circles. b. Understand properties of and differences between perpendicular and parallel lines. **MP.3, MP.6**

KY.HS.G.13 (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **MP.6, MP.7** Area of triangle = ½ ab sin(C)

KY.HS.G.14 (+) Understand and apply the Law of Sines and the Law of Cosines. a. Use the Law of Sines and Cosines to find unknown measurements in right and non-right triangles. b. Prove the Laws of Sines and Cosines and use them to solve problems. **MP.1, MP.3**

KY.HS.G.15 Verify using dilations that all circles are similar. **MP.5, MP.8**

KY.HS.G.17 (+) Apply basic construction procedures within the context of a circle. a. Construct the inscribed and circumscribed circles of a triangle. b. Construct a tangent line from a point outside a given circle to the circle. **MP.5, MP.6**

KY.HS.G.18 (+) Understand the relationship between an intercepted arc length within a circle and the circle's radius.
a. Derive using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector.
b. Define the radian measure of the angle as the measure of a central angle that intercepts an arc equal in length to the circle's radius. **MP.2, MP.3**

KY.HS.G.21 Use coordinates to justify and prove simple geometric theorems algebraically. **MP.2, MP.6**

KY.HS.G.29 Use geometric shapes, their measures and their properties to describe objects in real world settings. **MP.1, MP.4**

Essential Vocabulary, Theorems, and Postulates

Essential Vocabulary by Topic	Essential Theorems/Postulates by Topic
Circle Vocabulary: central angle, chord, inscribed angle, intercepted arc, major arc, minor arc, semicircle, secant, tangent to a circle; circumscribed figure, inscribed figure, central angle, arc, point of tangency, tangent line, secant line, equation of circle, sector	•Tangent-Radius Theorem (tangent is perpendicular to the radius at the point of contact) •Two-Tangent Theorem (two tangents meeting an external point are congruent) •Congruent chords --> congruent arcs --> congruent central angles •Chords equidistant from the center are congruent •If a diameter is perpendicular to a chord, then it bisects the chord •Inscribed Angles Theorem and corollaries •Angle-arc relationships for all angles with vertices on the center, on the circle, inside the circle but not on the center, and outside the circle •"Power Theorems" (relating segment lengths of chords, secants, and tangent segments of a circle)
Triangle and Trig Vocabulary: Pythagorean Theorem, radical, angle of depression, angle of elevation, 30-60-90 triangle, 45-45-90 triangle, trigonometry, sine, cosine, tangent, Pythagorean triple, trigonometric ratios	**Triangle Theorems/Postulates:** •The Pythagorean Theorem and its Converse •Ratios of sides in 30°-60°-90° special right triangles •Ratios of sides in 45°-45°-90° special right triangles •SOH-CAH-TOA

Benchmark Assessment #2

Anchor Resources

enVision Topic 8 - Right Triangles and Trigonometry	enVision Topic 10 Circles
MILC - MILC Topic Right Triangles and Trigonometry (11 days) resources **FAL -** Proving the Pythagorean Theorem **3 ACT Math Task** - The Impossible Measurement (after 8-4) Include simplifying single radicals, operations with radicals, and rationalizing the denominator **STEM** Activity: "Measure a Distance" (can be completed around ACT	**MILC -** MILC Topic 10 **Circles resources** **FAL** (one per semester): FAL Equations of Circles 1.pdf ** Recommended 2nd-semester Geometry FAL** Desmos to use before starting Topic 10 OR before FAL - **Intro: Equations of Circles • Activity Builder by Desmos**

date)	

(**Statistics Review posted on MILC and also highlight Algebra standards in a skills review - include quadratics**)

***Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.
** Mathematical Practices (A.MP.1- 8) should be evidenced throughout each unit, depending on the explored tasks. It is important to note that MP. 2 should support learning in every lesson.
*** Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.