

Unit 4 Title	Estimated Time Frame
Probability and Geometry	40 days
Big Idea	
Investigate chance processes and develop, use, and evaluate probability models. Draw, construct and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	
Essential Question(s)	
How can a model help me solve a probability problem? How can you investigate chance processes and develop, use and evaluate probability models? How can geometry be used to solve problems? How is the circumference of a circle used to derive the area of a circle? How are area and volume properties related?	
Standards for Mathematical Practice (MP.) - The practice standards in bold describe expertise to be intentionally developed in this unit.	Kentucky Interdisciplinary Literacy Practices (KILP.) - The practice standards in bold describe expertise to be intentionally developed in Mathematics.
MP.1. Make sense of problems and persevere in solving them. MP.2. Reason abstractly and quantitatively. MP.3. Construct viable arguments and critique the reasoning of others. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure. MP.8. Look for and express regularity in repeated reasoning.	KILP.1 Recognize that text is anything that communicates a message. KLIP.2 Employ, develop, and refine schema to understand and create text. KILP.3 View literacy experiences as transactional, interdisciplinary and transformational. KILP.4 Utilize receptive & expressive language arts to better understand self, others, and the world. KILP.5 Apply strategic practices, with scaffolding & then independently, to approach new literacy tasks. KILP.6 Collaborate with others to create new meaning. KILP.7 Utilize digital resources to learn and share with others. KILP.8 Engage in specialized, discipline specific literacy practices. KILP.9 Apply high level cognitive processes to think deeply and critically about text. KILP.10. Develop a literacy identity that promotes lifelong learning.
Common Preconceptions/Misconceptions	
Student thinking about theoretical probability is extended to developing a model (MP.4) that lends structure (MP.7) to an otherwise abstract idea. Students may use this model to explain why a penny comes up heads half the time and tails the other half; however, in an experiment where this event is repeated multiple times, the experimental probability may not be exactly $\frac{1}{2}$ and $\frac{1}{2}$. (MP.8).	

Compound probability may be more difficult for students to understand; tree diagrams, lists, etc., may help students understand the concept. Difficult-to-understand compound events may necessitate a simulation tool, for example, a random digit generator.

Initially, students may struggle with moving from a concrete understanding of a real-world situation to a miniature version, or vice versa; hands-on measurements and the use of technology can assist students with this abstract idea.

Students may confuse vocabulary words introduced in 7.G.5 (*supplementary*, *complementary*, *vertical*, and *adjacent*.) Having students make a foldable where they can make the correct distinction can be helpful.

Students may mischaracterize the volume and surface area of three-dimensional shapes, leading them to develop ways to decide whether a situation calls for the volume of a figure, or the surface area of a figure. Using nets and other appropriate tools gives students a structure to foster a greater understanding of the surface area.

KAS Standards	Prerequisite Skill, Considerations, and Coherence	Samples of Learning Intentions and Success Criteria
Cluster: Investigate chance processes and develop, use and evaluate probability models.		
<p>KY.7.SP.5 Describe the probability of a chance event as a number between 0 and 1, which tells how likely the event is, from impossible (0) to certain (1). A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</p> <p><input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application</p> <p>MP.5, MP.6, MP.7, KILP.1, KILP.8</p>	<p>Considerations: Emphasis is on the descriptive language used to describe numerical probabilities; impossible event, unlikely event, equally likely event, likely event, and certain event. Students understand all probabilities must fall between 0 and 1.</p>	<p>We are learning to understand likelihood and probability.</p> <ul style="list-style-type: none"> • I can use a number between 0 and 1 to describe the likelihood that an event will occur. • I can use a word: impossible, unlikely, equally likely, likely, or certain to describe the likelihood that an event will occur. • I can calculate the probability that an event will occur.
<p>KY.7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it, observing its long-run relative frequency, and predicting the approximate relative frequency given the probability.</p> <p><input type="checkbox"/> Conceptual <input type="checkbox"/> Procedural <input type="checkbox"/> Application</p> <p>MP.1, MP.2, KILP.2, KILP.6, KILP.7</p>	<p>Considerations: Estimate the likelihood of an event, test the estimate by trial, and collect data. Students observe the accuracy of the estimate will increase with the frequency of repeated trials.</p>	<p>We are learning to use the experimental probability of an event to solve real-world problems.</p> <ul style="list-style-type: none"> • I can find the theoretical probability of an event. • I can find the experimental probability of an event. • I can use experimental probability to predict the approximate relative frequency.

KY.7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

☐ **Conceptual** ☐ **Procedural** ☐ **Application**

a. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events.

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

MP.4, MP.7, MP.8, KILP.2, KILP.4, KILP.7

Considerations:

a. If a student is selected randomly from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

b. Find the approximate probability a spinning penny will land heads up, or a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Prerequisite Skills: KY.7.RP.3

We are learning to develop a probability model and use it to find probabilities of events then compare the model to observed frequencies.

- I can compare the experimental and theoretical probabilities of an event.
- I can determine the total number of possible outcomes for an event and use it to find the probability of a particular outcome.
- I can create a probability model with a sample space and list of events.
- I can use the probability model to determine the probabilities of events.

KY.7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

☐ **Conceptual** ☐ **Procedural** ☐ **Application**

a. Explain just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

☐ **Conceptual** ☐ **Procedural** ☐ **Application**

b. Represent sample spaces for compound events described in everyday language using organized lists, tables, and tree diagrams.

☐ **Conceptual** ☐ **Procedural** ☐ **Application**

c. Design and use a simulation to generate frequencies for compound events.

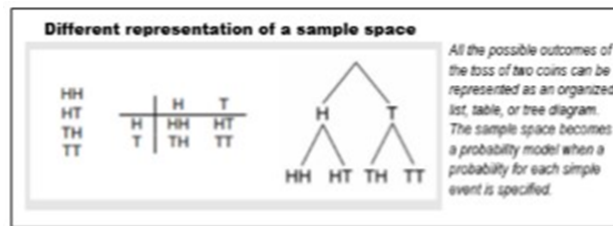
☐ **Conceptual** ☐ **Procedural** ☐ **Application**

MP.2, MP.4, MP.7, KILP.5, KILP.7, KILP.8

Considerations:

a. If the probability of heads occurring on a coin is $\frac{1}{2}$, then the probability of three heads in a row is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

b. For a simulation of tossing two fair coins:



We are learning how to determine the outcomes of compound events.

- I can write the compound probability as a number between 0 and 1.

We are learning about probabilities of compound events.

- I can simulate a compound event to generate a sample.
- I can use a tree diagram, a table, or an organized list to represent the sample space for a compound event.
- I can find the probability of a compound event using sample space models.

Cluster: Draw, construct and describe geometrical figures and describe the relationships between them.

KY.7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

☐ Conceptual ☒ **Procedural** ☐ Application

MP.1, MP.2, MP.5, KILP.5, KILP.8

Considerations: Emphasize converting values from one given measurement to another based on a given scale factor. For example, 1 inch on the scale drawing equals how many feet in real life based on the scale factor given. Students reproduce a given drawing based on a scale factor.

Coherence KY.6.G.1→KY.7.G.1→KY.8.EE.6

We are learning to use scale drawings of geometric figures.

- I can find actual lengths using a scale drawing.
- I can use scale factors to solve area problems.
- I can convert a scale drawing to a different scale.
- I can reproduce a drawing based on a different scale factor.

KY.7.G.2 Draw (freehand, with ruler and protractor and with technology) geometric shapes with given conditions.

☐ Conceptual ☒ **Procedural** ☐ Application

MP.6, MP.7, KILP.2, KILP.7

Considerations: Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Coherence KY.7.G.2→KY.8.G.1

We are learning to construct geometric figures.

- I can construct triangles from three angle measures using a ruler, protractor, or technology given certain conditions.
- I can construct triangles from three side measures using a ruler, protractor, or technology given certain conditions.
- I can determine the number of triangles that can be formed given certain side lengths and angle measures.

Cluster: Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

KY.7.G.5 Apply supplementary, complementary, vertical, and adjacent angles properties in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

☐ Conceptual ☒ **Procedural** ☐ Application

MP.3, MP.6, MP.7, KILP.2, KILP.8

Considerations: Emphasis is on the relationships between the various angles listed to find missing angles based on the relationships and to write and solve equations to find unknown angles.

Coherence KY.4.MD.7 →KY.7.G.5 →KY.8.G.5

We are learning to use angle relationships to solve problems.

- I can apply properties of supplementary, complementary, vertical, and adjacent angles to find the measures of missing angles.
- I can apply the properties of angles to write and solve equations.
- I can substitute a variable value in to find the angle measure.
- I can recognize the relationship between angles formed by intersecting lines and rays.

KY.7.G.4 Use formulas for the area and circumference of circles and their relationships.

☐ **Conceptual** ☐ **Procedural** ☐ **Application**

a. Apply the formulas for the area and circumference of a circle to solve real-world and mathematical problems.

b. Explore and understand the relationship between a circle's radius, diameter, circumference, and area.

MP.1, MP.2, MP.8, KILP.2, KILP.5, KILP.8

Supporting Standard [KY.7.EE.3](#)

Considerations:

Circle Formulas: $C = d\pi$ $C = 2r\pi$ $A = \pi r^2$

Note: Calculating the radius or diameter of a circle given its area is not expected, as finding the square root of a number is reserved for 8th grade.

a. Both area and circumference are represented; students recognize when the circumference is needed and when the area is needed.

b. Emphasis is on calculating the area given diameter, finding the circumference given radius or diameter, and finding the radius or diameter given circumference. Special attention is given to the relationship between diameter and circumference as a ratio that leads to pi.

Coherence KY.7.G.4 → KY.8.G.9

We are learning to solve problems involving the circumference of a circle.

- I can use the formula *Circumference* = πd or $2\pi r$ to calculate the circumference of a circle.
- I can determine the radius or diameter using the circumference formula.
- I can identify the relationship between the circumference and diameter of a circle.

We are learning to solve problems involving the area of a circle.

- I can use the formula *Area* = πr^2 to calculate a circle's area when given diameter or radius.

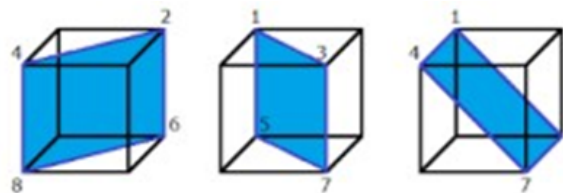
Cluster: Draw, construct and describe geometrical figures and describe the relationships between them.

KY.7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

☐ **Conceptual** ☐ **Procedural** ☐ **Application**

MP.5, MP.6, KILP.7, KILP.8

Considerations: Cross sections may be taken from horizontal, vertical and oblique angles, such as



We are learning to describe cross-sections of three-dimensional figures.

- I can describe the 2-dimensional figure formed when slicing a right rectangular prism or pyramid horizontally, vertically, or diagonally (cross-section).

Cluster: Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

KY.7.G.6 Solve problems involving the area of two-dimensional objects and the surface area and volume of three-dimensional objects.

☐ Conceptual ☒ **Procedural** ☐ **Application**

a. Solve real-world and mathematical problems involving the area of two-dimensional objects composed of triangles, quadrilaterals, and other polygons.

b. Solve real-world and mathematical problems involving volume and surface area, using nets of three-dimensional objects, including cubes, pyramids, and right prisms.

MP.3, MP.4, MP.5, KILP.1, KILP.3, KILP.9

Supporting Standard [KY.7.EE.3](#)

Considerations:

- Emphasis is on finding the area of composite figures composed of convex polygons.
- Students understand volume and surface area are two different quantities used to describe the same three-dimensional figure. Building upon their understanding of area, students use nets of three dimensional objects to conceptualize surface area. Students calculate with appropriate units, using nets as a possible strategy for calculation as well as formulas for volume and surface area, where appropriate.

Coherence KY.6.G.4 → KY.7.G.6 → KY.8.G.6

We are learning to solve problems involving the area of two-dimensional figures and surface area.

- I can find the area of composite 2-dimensional figures by finding the sum of the areas of each part.
- I can find the surface area of a 3-dimensional figure by finding the sum of areas of each part of its net.

We are learning to solve problems involving the volume of three-dimensional objects.

- I can use the area of the base and height of a 3-dimensional figure to find its volume.
- I can find the missing dimension when given the volume of a figure.

Essential Vocabulary

chance event - Anything that happens suddenly or by chance without an apparent cause, ex: Winning the lottery.

compound event - An event whose probability of occurrence depends upon the probability of occurrence of two or more independent events. An event that consists of two or more events that are not mutually exclusive.

probability - A number between 0 and 1 is used to quantify the likelihood of processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

probability model - A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

random sample - A sample in which every element in the population has an equal chance of being selected.

theoretical probability - The probability/likelihood of an event happening based upon mathematical calculations: $P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{total number of possible outcomes}}$.

adjacent angles - Two angles that share both a side and a vertex.

angle - The union of two rays with a common endpoint, called the vertex.

area - The number of square units that covers a shape or figure.

circle - A closed curve with all its points in one plane and the same distance from a fixed point, the center.

circumference - Distance around a circle; its perimeter.

complementary - Two angles whose sum is 90 degrees.

congruence - Two plane or solid figures are congruent if they have the same size and shape.

cross-section - The intersection of a 3-dimensional body with a plane.

radius - A line segment drawn from the center of a circle to any point on a circle; half the diameter.

scale drawing - A drawing that is a reduction or enlargement of the original.

scale factor - A number that multiplies some quantity; the ratio of any two corresponding lengths in two similar geometric figures.

supplementary - Two angles are supplementary if their sum is 180 degrees.

surface area - For a three-dimensional figure, the sum of the areas of all the faces.

Supporting Standards

[KY.7.EE.3](#) Solve real-life and mathematical problems posed with positive and negative rational numbers in any form, using tools, strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. **MP.1, MP.4, MP.6**

Considerations: Students solve multi-step real-world and mathematical problems containing integers, fractions, and decimals, using previously acquired skills around converting fractions, decimals, and percentages and using properties of operations to find equivalent forms of expressions when needed. Students solidify understanding by checking their solutions for reasonableness using estimation strategies such as rounding, compatible numbers and benchmark numbers.

☐ Conceptual ☐ Procedural ☐ Application

Benchmark Assessment

Resources

Standard Resource Pages Hyperlinked to Each Standard

[enVision Crosswalk Unit 4](#)

enVision Language Support Handbook

[Three Reads Routine](#)

[Notice and Wonder Routine](#)

[MILC Resources](#)

*Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

** Mathematical Practices (A.MP.1- 8) should be evidenced at some point throughout each unit.

*** Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.