Unit 3 Title	**Estimated Time Frame**
Rational Exponents, Radical Functions, and **Probability**	**45 days** or **22 block days**

Big Idea (s)

Students will extend their previous understanding of radical functions.
Students will use properties of rational exponents and radicals.
Students will learn methods to graph radical functions, solve radical equations, and combine functions.
Students will identify inverses of functions and learn to write the equations of inverse functions.

Students will extend their previous understanding of ratios and basic probability to the probability of multiple events, combinatorics, probability distributions, and expected value.
Students will understand and graph probability distributions.
Students will use probability models and expected values to make decisions.

Essential Question(s)

How are rational exponents and radical equations used to solve real-world problems?
For what input values is the output value positive, negative, or 0?
What happens to the output when the input value gets very large in magnitude?"
How can you find the probability of events and combinations of events?

Standards for Mathematical Practice (MP.) - The practice standards in bold describe expertise to be intentionally developed in this unit.	**Kentucky Interdisciplinary Literacy Practices (KILP.) -** The practice standards in bold describe expertise to be intentionally developed in Mathematics.
MP.1. Make sense of problems and persevere in solving them. **MP.2. Reason abstractly and quantitatively.** **MP.3. Construct viable arguments and critique the reasoning of others.** **MP.4. Model with mathematics.** **MP.5. Use appropriate tools strategically.** MP.6. Attend to precision. **MP.7. Look for and make use of structure.** **MP.8. Look for and express regularity in repeated reasoning.**	KILP.1 Recognize that text is anything that communicates a message. KLIP.2 Employ, develop, and refine schema to understand and create text. KILP.3 View literacy experiences as transactional, interdisciplinary and transformational. KILP.4 Utilize receptive & expressive language arts to better understand self, others, and the world. KILP.5 Apply strategic practices, with scaffolding & then independently, to approach new literacy tasks. **KILP.6 Collaborate with others to create new meaning.** **KILP.7 Utilize digital resources to learn and share with others.** **KILP.8 Engage in specialized, discipline specific literacy practices.** **KILP.9 Apply high level cognitive processes to think deeply and critically about text.** KILP.10.Develop a literacy identity that promotes lifelong learning.

Common Preconceptions/Misconceptions

Students often solve rational and radical equations without checking that their solutions may be erroneous. One method that helps students understand extraneous roots is to look at the graph of the original function associated with the equation. Students need to check their own work and determine if a possible solution does or does not work.

Students will need examples to show that association does not necessarily provide evidence for cause and effect. Students often have difficulty separating the ideas of association and casualty.

Students work with set notation to represent multiple contexts. Allowing time for students to grapple with the connection of these ideas can help them develop fluency with notation and connecting notation to multiple problem situations.

KAS Standards	Considerations	Samples of Learning Intentions and Success Criteria
Cluster: Understand solving equations as a process of reasoning and explain the reasoning.		
KY.HS.A.17 Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise. **b.** Solve radical equations in one variable. **MP.3, MP.5, MP.7** *Supporting Standard(s): KY.HS.A.15, KY.HS.A.16* ☐ Conceptual ☐ Procedural ❑ Application	Students analyze solution sets of equations to determine processes (for example, squaring both sides of an equation) that might lead to a solution set that differs from the original equation.	We are learning to solve radical equations with one variable. ● I can square both sides of the equation to eliminate the radical and solve for the variable, ● I can justify solutions and recognize when extraneous solutions may arise. ● I can substitute solutions back into the original equation to verify that they satisfy the equation.
Cluster: Understand the concept of a function and use function notation.		
KY.HS.F.1 Understand properties and key features of functions and the different ways functions can be represented. **c.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. **MP.2, MP.4, MP.7, KILP.1, KILP.2, KILP.7**	A function is often described and understood in terms of the output behavior or over what input values it is increasing, decreasing, or constant. Important questions include, "For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large in magnitude?" Graphs become useful representations for understanding and comparing functions because these behaviors are often easy to see in the graphs of functions.	We are learning to graph and interpret key features of graphs of radical functions. ● I can identify the domain and range of a radical function. ● I can sketch the graphs of a radical function. We are learning to interpret key features of graphs and tables of functions that model relationships between two quantities and sketch the graph. ● I can sketch graphs that accurately represent the key features described

Supporting Standard(s): KY.HS.A.2, KY.HS.N.1, & KY.HS.N.2 ☐ Conceptual ❑ Procedural ❑ Application	**Key features include** but are not limited to intercepts, intervals where the function is increasing, decreasing, or remaining constant; relative maxima and minima; symmetries; end behavior, and periodicity.	verbally, including shape, direction, and position of important points. ● I can interpret key features of graphs and tables given a verbal description.
Cluster: Build new functions from existing functions.		
KY.HS.F.8 Understand the effects of transformations on the graph of a function. a. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx) and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. ☐ Conceptual ☐ Procedural ❑ Application b. Experiment with cases and explain the effects on the graph using technology. **MP.3, MP.5, KILP.5, KILP.8, KILP.9** *Supporting Standard(s): KY.HS.A.1, KY.HS.F.3.b* ☐ Conceptual ☐ Procedural ☐ Application	Mastery of Part a of this standard includes recognizing even and odd functions from their graphs and algebraic expressions.	We are learning about the effects of transformations on the graph of a function. ● I can understand the types of transformations that can be applied to a function graph, including translations, reflections, stretches, and compressions. ● I can understand how translations affect the graph horizontally and vertically, shifting the function left, right, up, or down. ● I can comprehend how reflections across the x-axis and y-axis change the orientation of the graph. ● I can identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. ● I can use technology to support my understanding and explanations of transformation effects.
Cluster: Build new functions from existing functions.		
KY.HS.F.9 Find inverse functions. a. Given the equation of an invertible function, find the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	a. Students can complete the process of finding the inverse when given an equation of a function that is invertible. b-d. Students need a formal sense of inverse functions. Students understand a function and its inverse describe the exact same relationship but in different ways.	We are learning to find the inverse functions when provided with the equation of an invertible function. ● I can Identify an invertible function and understand that it has a unique inverse function. ● I can apply algebra to find the inverse of a given invertible function, such as solving for the variable in terms of the output or

d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **MP.2, MP.6** *Supporting Standard(s):* ☐ Conceptual ☐ Procedural ☐ Application		interchanging the roles of input and output variables. ● I can verify the correctness of the inverse function by composing it with the original function and observing that the result is the identity function. ● I can graphically represent the inverse function by reflecting the graph of the original function across the line y = x. ● I can apply the inverse function to solve equations and real-world problems involving the original function.
Cluster: Summarize, represent and interpret data on a single count or measurement variable.		
KY.HS.SP.2 Use statistics appropriate to describe the shape of the numerical data distribution to compare center (median, mean) and spread (interquartile range when comparing medians and standard deviation when comparing means) of different data distributions. **MP.2, MP.6, KILP.1, KILP.2, KILP.7** *Supporting Standard(s):KY.HS.SP.1, KY.HS.SP.10, KY.HS.SP.11* ☐ Conceptual ❑ Procedural ☐ Application	Students use raw data and data from appropriate graphical representations to compare differences in the shape, center, spread and presence of outliers and other unusual features of comparable data sets.	We are learning to analyze numerical data distributions using appropriate statistics to compare their centers (median, mean) and spreads (interquartile range for comparing medians and standard deviation for comparing means). ● I can determine the appropriate measures of center and spread for a distribution and use them to compare distributions.
Cluster: Understand independence and conditional probability and use them to interpret data.		
KY.HS.SP.14 Describe events as subsets of a sample space. Use characteristics (or categories) of the outcomes, such as unions, “A or B,” that are mutually exclusive events, unions, “A or B,” that are non-mutually exclusive events, and as intersections, “A and B,” and as	A union of two events, “A or B,” includes all elements in both events notated by: $A \cup B$. Addition Rule for mutually exclusive events: If A and B are mutually exclusive, P(A or B) = P(A) + P(B). Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. An intersection, “A and B,”	We are learning to calculate probabilities of multiple events. ● I can define unions. intersections, and complements of events and calculate probabilities of these events. ● I can describe events as subsets of a sample space using characteristics of the

complements of other events, "not A." to calculate basic probabilities. **MP.1, MP.2, KILP.1, KILP.6, KILP.7** *Supporting Standard(s): KY.HS.SP.12, KY.HS.SP.13* ☐ Conceptual ☐ Procedural ☐ Application	of two events include all overlapping elements notated by: $A \cap B$. A complement for any event A, P(not A) = 1 – P(A).	outcomes ("or," "and," "not")
KY.HS.SP.15 Understand the concept of independence. **a.** Understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their individual probabilities, P(A) x P(B) **c.** Recognize and explain the concept of independence in everyday language and everyday situations. **MP.1, MP.6, KILP.1, KILP.3, KILP.9** *Supporting Standard(s): KY.HS.SP.10* ☐ Conceptual ☐ Procedural ☐ Application	a. Events A and B are independent if and only if P(A and B) = P(A)P(B).	We are learning to understand the concept of independence. ● I can categorize events as independent or not using the characterization that two events, A and B, are independent when the probability of A and B occurring together is the product of their probabilities. ● I can determine the outcome of independent events as the product of their probabilities.
KY.HS.SP.16 Understand the concept of conditional probability. a. Understand the conditional probability of A given B as P(A and B)/P(B). c. Recognize and explain the concept of conditional probability in everyday language and everyday situations. d. Find the conditional probability of A given B as the fraction of B's outcomes belonging to A and interpret the answer in terms of the model. **MP.1, MP.3, KILP.1, KILP.3, KILP.9**	For any two events A and B, P(A given B) = $\frac{P(A \text{ and } B)}{P(B)}$.	We are learning to understand, recognize, and explain the concept of conditional probability. ● I can understand the conditional probability of A given B as P(A and B)/P(B). ● I can interpret the independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

Supporting Standard(s): KY.HS.SP.5 ☐ Conceptual ☐ Procedural ☐ Application		
Cluster: Use the rules of probability to compute probabilities of compound events		
KY.HS.SP.19 Use permutations and combinations to compute probabilities. a. Distinguish between situations that can be modeled using counting techniques, including the Fundamental Counting Principle, permutations, and combinations. b. Perform calculations using the appropriate counting technique, including simple probabilities. **MP.1, MP.8, KILP.1, KILP.2, KILP.7** *Supporting Standard(s): KY.HS.N.5, KY.HS.N.6* ☐ Conceptual ☐ Procedural ☐ Application	Permutations are calculated when order matters. Combinations are calculated when order does not matter. Number of permutations of n items taken r at a time: ${}_nP_r = \frac{n!}{(n-r)!}$ Number of combinations of n items taken r at a time: ${}_nC_r = \frac{n!}{(n-r)!r!}$	We are learning to use permutations and combinations to compute probabilities of compound events and solve problems. ● I can identify situations that are permutations and those that are combinations. ● I can use permutations to compute probabilities of compound events and solve problems. ● I can use combinations to compute probabilities of compound events and solve problems.
Cluster: Perform operations on matrices and use matrices in applications.		
KY.HS.N.14 Use matrices to represent and manipulate data. **MP.4, MP.5** *Supporting Standard(s): KY.HS.N.5, KY.HS.N.6, KY.HS.N.15* ☐ Conceptual ☐ Procedural ☐ Application	Students understand matrices are rectangular arrays comprised of elements that are useful for solving problems in context.	We are learning to use matrices to represent and manipulate data, understanding their applications in various contexts ● I can understand the concept of matrices and their role in organizing and representing data in a structured format. ● I can identify situations where matrices can be used to represent data, such as in systems of linear equations, transformations, or statistical analysis. ● I can represent real-world datasets as matrices, ensuring that the dimensions of the matrices match the dimensions of the data.

Supporting Standards

KY.HS.N.1 Extend the properties of integer exponents to rational exponents, allowing for the expression of radicals in terms of rational exponents. **MP.2, MP.7**

KY.HS.N.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. **MP.7**

KY.HS.N.5 Define appropriate units in context for the purpose of descriptive modeling. ★ **MP.1, MP.6**

KY.HS.N.6 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities**.** ★ **MP.2, MP.6**

KY.HS.F.3.b. Estimate the rate of change from a graph**.** ★ **MP.2, MP.4**

KY.HS.A.1 interpret expressions representing a quantity in terms of its context. ★ **MP.2, MP.6**
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions, given a context, by viewing one or more of their parts as a single entity.

KY.HS.A.2 Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. **MP.7, MP.8**

KY.HS.A.15 Rearrange formulas to solve a literal equation, highlighting a quantity of interest, using the same reasoning as in solving equations. **MP.2, MP.7**

KY.HS.A.16 Understand each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **MP.1, MP.3**

KY.HS.N.15 Perform operations with matrices. a. Add, subtract and multiply matrices of appropriate dimensions. b. Multiply matrices by scalars to produce new matrices. **MP.7, MP.8**

KY.HS.SP.1 Represent the distribution of data with plots on the real number line (stem plots, dot plots, histograms and box plots). **MP.4, MP.5**

KY.HS.SP.4(+) When appropriate, fit a normal distribution to a numerical data set for a given mean and standard deviation and then estimate population percentages using the Empirical Rule and recognize that there are data sets for which such a procedure is not appropriate. **MP.1, MP.3**

KY.HS.SP.5 Summarize categorical data for two or more categories in frequency tables. Calculate and interpret joint, marginal and conditional relative frequencies (probabilities) in the context of the data, recognizing possible associations and trends in the data. **MP.2, MP.7**

KY.HS.SP.9 Understand statistics as a process for making inferences and justifying conclusions about population parameters based on a random sample from that population. **MP.1, MP.3**

KY.HS.SP.10 Decide if a specified model is consistent with the results from a simulation. **MP.3, MP.6**

KY.HS.SP.11 Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. **MP.3, MP.8**

KY.HS.SP.12 Use data from a sample survey to estimate a population mean or proportion and explain how bias may be involved in the process. **MP.4, MP.7**

KY.HS.SP.13 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between estimates or statistics are significant. **MP.3, MP.8**

KY.HS.SP.15. b (+) Determine whether two events are independent and provide a justification to support the decision.

KY.HS.SP.17 (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide whether events are independent and to approximate conditional probabilities. **MP.2, MP.4**

KY.HS.SP.18 (+) Apply the General Multiplication Rule, P(A and B) = P(A)P(B|A) = P(B)P(A|B), in a uniform probability model, and interpret the answer in terms of the model. **MP.1, MP.2**

KY.HS.SP. 19. c. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

KY.HS.SP.20 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same appropriate graphical displays as for data distributions**. MP.3, MP.6,**

KY.HS.SP.21 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution and use the value in analyzing decisions. **MP.1, MP.8**

KY.HS.SP.22 (+) Develop a probability distribution for a random variable. a. Find an expected value based on a sample space in which theoretical probabilities can be calculated. b. Find an expected value based on a sample space in which empirical probabilities can be calculated. **MP.2, MP.8**

KY.HS.SP.23 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
a. Find the expected payoff for a game of chance.
b. Evaluate and compare strategies based on expected values.
c. Use calculated expected values to make fair decisions and formulate strategies. **MP.6, MP.8**

Essential Vocabulary

Rational Exponents/Radical Functions:	Probability Vocabulary:
complex conjugate exponent Index nth root radical symbol radicand radical expression rational exponent domain composite function composition of functions dependent variable independent variable relation inverse function inverse relation	Event/Independent events Outcome Sample space Mutually exclusive Conditional probability Dependent events Combination Factorial Fundamental counting principle Permutation Binomial probability

Standards Benchmark Assessment #2

March 2025

Anchor Resources

enVision Topic 5 – Rational Exponents and Radical Functions	enVision Topic 12 Probability
MILC - **MILC Topic 5 Rational Exponents and Radical Functions resources** **FAL** (one per semester): **Evaluating Statements About Radicals**(use with 5-4 OR anytime after 5-4) ****Recommended FAL for 2nd semester)** **3 Act Math Task** - *The Snack Shack* - (3 kids on the beach go to the snack shack (paths) - Use after **Lesson 5-4**	**MILC -** **MILC Topic 12 resources** **3 Act Math Task - Free Throws for the Win!** (use the Calipari video clip)

*****Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

** Mathematical Practices (A.MP.1- 8) should be evidenced throughout each unit, depending on the tasks.

******* Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.