

<b>Unit 1 Title</b>		<b>Estimated Time Frame</b>
<b>Linear Functions and Systems &amp; Quadratic Functions and Equations</b>		<b>40 days OR 20 blocks</b>
<b>Big Idea (s)</b>		
<p>Students should be able to solve linear and quadratic equations algebraically and graphically.                  Students will apply these prerequisite skills to solve problems and analyze situations using one or more functions.                  Students will extend their previous understanding of quadratic functions.                  Students will identify different forms of quadratic functions and their key features.                  Students will explore complex numbers and solve problems with complex numbers.                  Students will learn different methods for solving quadratic equations.</p>		
<b>Essential Question(s)</b>		
<p>What are the ways in which functions can be used to represent and solve problems involving quantities?                  How do you use quadratic functions to model situations and solve problems?</p>		
<b>Standards for Mathematical Practice (MP.) -</b> The practice standards in bold describe expertise to be intentionally developed in this unit.	<b>Kentucky Interdisciplinary Literacy Practices (KILP.) -</b> The practice standards in bold describe expertise to be intentionally developed in Mathematics.	
MP.1. Make sense of problems and persevere in solving them. <b>MP.2. Reason abstractly and quantitatively.</b> <b>MP.3. Construct viable arguments and critique the reasoning of others.</b> <b>MP.4. Model with mathematics.</b> <b>MP.5. Use appropriate tools strategically.</b> MP.6. Attend to precision. <b>MP.7. Look for and make use of structure.</b> <b>MP.8. Look for and express regularity in repeated reasoning.</b>	KILP.1 Recognize that text is anything that communicates a message. KILP.2 Employ, develop, and refine schema to understand and create text. KILP.3 View literacy experiences as transactional, interdisciplinary and transformational. KILP.4 Utilize receptive & expressive language arts to better understand self, others, and the world. KILP.5 Apply strategic practices, with scaffolding & then independently, to approach new literacy tasks. <b>KILP.6 Collaborate with others to create new meaning.</b> <b>KILP.7 Utilize digital resources to learn and share with others.</b> <b>KILP.8 Engage in specialized, discipline specific literacy practices.</b> <b>KILP.9 Apply high level cognitive processes to think deeply and critically about text.</b> KILP.10. Develop a literacy identity that promotes lifelong learning.	
<b>Common Preconceptions/Misconceptions</b>		
<b>Prerequisite Skills:</b>		
<ul style="list-style-type: none"> <li>Equations, verbal descriptions, graphs, and tables</li> <li>Construct functions with and without technology</li> </ul>		

- Solve equations and inequalities
- Transform linear functions
- Absolute value

**Misconceptions:**

- Some students may find only one x-intercept. Have students check their work by substituting 0 for y and solving for x, so there are two x-intercepts.
- Students may forget when to use a bracket and when to use a parenthesis in interval notation. Have students make flashcards to review. One side should have interval notation, and the other side should represent the same interval on a number line. Students should focus on recognizing that a parenthesis in the interval notation corresponds to an open circle on the number line and a bracket represents a closed circle.
- Students may confuse about which way the inequality signs should face when finding the solution.
- Students may treat the coefficient of  $i$  as 0 instead of 1.

KAS Standards	Considerations	Samples of Learning Intentions and Success Criteria
<p><b>Cluster: Understand the concept of a function and use function notation.</b></p>		
<p><b>KY.HS.F.1</b> Understand properties and key features of functions and the different ways functions can be represented.</p> <p>a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>.</p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p> <p>b. Using appropriate function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.</p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p> <p>c. For a function that models a relationship</p>	<p>A function is often described and understood in terms of the output behavior, or over what input values is it increasing, decreasing, or constant. Important questions include, “For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large in magnitude?” Graphs become useful representations for understanding and comparing functions because these behaviors are often easy to see in the graphs of functions.</p> <p><b>Key features include but are not limited to:</b> intercepts; intervals where the function is increasing, decreasing, or remaining constant; relative maxima and minima; symmetries; end behavior; periodicity</p>	<p>We are learning how to interpret key features of functions.</p> <ul style="list-style-type: none"> <li>• I can recognize the definition of a function and understand its role in modeling relationships between two quantities.</li> <li>• I can identify key features of graphs and tables representing functions, including intercepts, slope, maxima, minima, and points of inflection.</li> <li>• I can interpret key features of graphs and tables in the context of the quantities being modeled, understanding how changes in one quantity affect the other.</li> </ul> <p>We are learning how the domain relates to the quantitative relationship described by the function.</p> <ul style="list-style-type: none"> <li>• I can understand the concept of the domain of a function as the set of all possible input values.</li> <li>• I can determine which input values are valid</li> </ul>

<p>between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship.</p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p> <p><b>d.</b> Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.  <b>MP.2, MP.4, MP.7, KILP.1, KILP.2, KILP.6</b></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p> <p><i>Supporting Standard(s):</i> <a href="#">KY.HS.F.4</a></p>		<p>for the function in the domain.</p> <ul style="list-style-type: none"> <li>I can relate the domain of a function to the specific quantitative relationship it describes, recognizing how the domain constraints reflect real-world constraints or conditions.</li> </ul>
---	--	---

<b>Cluster: Build new functions from existing functions.</b>	
--	--

<p><b>KY.HS.F.8</b> Understand the effects of transformations on the graph of a function.</p> <p>a. Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs.</p> <p>b. Experiment with cases and illustrate an explanation of the effects on the graph using technology.  <b>MP.3, MP.5, KILP.5, KILP.7, KILP.9</b></p> <p><i>Supporting Standard(s):</i> <a href="#">KY.HS.F.4</a></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>a. Mastery of this standard includes recognizing even and odd functions from their graphs and algebraic expressions.</p>	<p>We are learning about the effects of transformations on the graph of a function.</p> <ul style="list-style-type: none"> <li>I can understand the types of transformations that can be applied to a function graph, including translations, reflections, stretches, and compressions.</li> <li>I can understand how translations affect the graph horizontally and vertically, shifting the function left, right, up, or down.</li> <li>I can comprehend how reflections across the x-axis and y-axis change the orientation of the graph.</li> <li>I can identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs.</li> <li>I can use technology to support my understanding and explanations of transformation effects.</li> </ul>
--	---	--

<b>Cluster: Represent and solve equations and inequalities graphically.</b>	
---	--

<p><b>KY.HS.A.24</b> Justify that the solutions of the equations <math>f(x) = g(x)</math> are the x-coordinates of the points where the graphs of <math>y = f(x)</math> and <math>y = g(x)</math> intersect. Find the approximate solutions graphically using technology or tables. ★  <b>MP.3, MP.5, KILP.6, KILP.7, KILP.9</b></p> <p><i>Supporting Standard(s):</i> <a href="#">KY.HS.A.17</a></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>Students justify solutions for equations that Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p>	<p>We are learning how to justify the solutions of the equations.</p> <ul style="list-style-type: none"> <li>● I can use graphs and graphing technology to find or approximate solutions to equations.</li> <li>● I can determine the solutions to equations using graphs and graphing technology.</li> <li>● I can justify the solutions of the equations <math>f(x) = g(x)</math> are the x-coordinates of the points where the graphs of <math>y = f(x)</math> and <math>y = g(x)</math> intersect.</li> </ul>
<p><b>Cluster: Interpret the structure of expressions.</b></p>		
<p><b>KY.HS.A.2</b> Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms.  <b>MP.7, MP.8, KILP.2, KILP.6</b></p> <p><i>Supporting Standard(s):</i> <a href="#">KY.HS.A.3</a>, <a href="#">KY.HS.A.7</a></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>Students see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, recognizing it as a difference of squares factored as <math>(x^2 - y^2)(x^2 + y^2)</math>. Additionally, students see there are three commonly used forms for a quadratic expression:</p> <p><b>Standard form, Factored form, Vertex form</b> and can identify when one form might be more useful than another</p>	<p>We are learning to use the structure of an expression to identify ways to rewrite it in equivalent forms.</p> <ul style="list-style-type: none"> <li>● I can determine key features of a quadratic function</li> <li>● I can write an equation for a parabola given a graph.</li> <li>● I can write quadratic equations in standard form.</li> <li>● I can analyze the structure of an expression to identify opportunities for rewriting it in simpler or more useful forms.</li> </ul>
<p><b>Cluster: Solve equations and inequalities in one variable.</b></p>		
<p><b>KY.HS.A.19</b> Solve quadratic equations in one variable.  a. Solve quadratic equations by taking square roots, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.  b. (+) Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.</p>	<p>Students observe that methods for solving quadratic equations are interrelated and certain situations may more appropriately call upon one method as opposed to the other methods. Students understand completing the square involves factoring and the quadratic formula is nothing more than an encapsulation of the method of completing the square. While all students are not required to be able to use completing the square as a method for solving quadratic equations, exposure to this method is needed to explain how the quadratic formula is derived.</p>	<p>We are learning how to solve the quadratic equations.</p> <ul style="list-style-type: none"> <li>● I can solve quadratic equations using the quadratic formula.</li> <li>● I can Identify the number of real solutions</li> <li>● I can Interpret the discriminant</li> <li>● I can solve quadratic equations by: <ul style="list-style-type: none"> <li>○ factoring</li> <li>○ taking square roots</li> <li>○ the quadratic formula</li> <li>○ (+) completing the square</li> </ul> </li> <li>● I can recognize the number and type of</li> </ul>

<p>c. (+) Solve quadratic equations by completing the square. <b>MP.1, MP.8, KILP.6, KILP.7</b></p> <p><i>Supporting Standard(s): <a href="#">KY.HS.A.3</a>, <a href="#">KY.HS.A.7</a>, <a href="#">KY.HS.A.13</a>, <a href="#">KY.HS.A.17</a></i></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>		<p>solutions.</p> <ul style="list-style-type: none"> <li>I can recognize when the quadratic formula gives complex solutions.</li> </ul>
<p><b>Cluster: Perform arithmetic operations with complex numbers.</b></p>		
<p><b>KY.HS.N.7</b> Understanding properties of complex numbers.</p> <p>a. Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math>, and <math>b</math> real.</p> <p>b. Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract and multiply complex numbers.</p> <p>c. (+) Find the conjugate of a complex number and use it to find the quotient of complex numbers. <b>MP.7, MP.8, KILP.3, KILP.8</b></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>a. Students understand that the complex number system provides solutions to the equation <math>xx^2 + 1 = 0</math> and higher-degree equations.</p> <p>c. Students understand the complex conjugate as the pair of binomial complex factors, <math>(a + bi)</math> <math>(a - bi)</math>, whose product is a difference of squares: <math>a^2 + b^2</math>, which is a real number. Students understand that the denominator of a fraction can be resolved of an imaginary number by multiplying by both the numerator and the denominator by the conjugate of the denominator</p>	<p>We are learning how to use properties of complex numbers to solve equations.</p> <ul style="list-style-type: none"> <li>I can understand the concept of complex numbers and understand their representation in the form <math>a + bi</math>.</li> <li>I can use the relation <math>i^2 = -1</math> and its relationship with real numbers and the complex number system.</li> <li>I can write the square root of a negative number in terms of <math>i</math>.</li> <li>I can perform operations with complex numbers.</li> </ul>
<p><b>Cluster: Use complex numbers in polynomial identities and equations.</b></p>		
<p><b>KY.HS.N.9</b> Solve quadratic equations with real coefficients that have complex solutions. <b>MP.1, MP.2, KILP.2, KILP.7, KILP.9</b></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>Students use the Quadratic Formula to solve for complex solutions. Students recognize that when a quadratic equation yields complex solutions its graph does not cross the x-axis.</p>	<p>We are learning how to solve equations with complex solutions.</p> <ul style="list-style-type: none"> <li>I can solve quadratic equations using the quadratic formula</li> <li>I can identify the number of real solutions</li> <li>I can Interpret the discriminant</li> <li>I can recognize that a quadric equation with complex solutions will have a graph that does not cross the x-axis.</li> </ul>
<p><b>Cluster: Solve systems of equations.</b></p>		

<p><b>KY.HS.A.21</b> Solve a simple system consisting of a linear equation and a quadratic equation in two variables, algebraically and graphically. <b>MP.3, MP.6, KILP.6, KILP.7</b></p> <p><i>Supporting Standard(s):</i> <a href="#">KY.HS.A.3</a> &amp; <a href="#">KY.HS.A.20</a>, <a href="#">KY.HS.A.22(+)</a>, <a href="#">KY.HS.N.14</a>, <a href="#">KY.HS.N.15</a></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>Students utilize algebra techniques and graphical representations to determine points of intersection between lines and parabolas that indicate solution sets for a system of linear and quadratic equations.</p> <p>A linear quadratic system consists of a linear equation and a quadratic equation. The points of intersection are the solutions.</p>	<p>We are learning how to find solutions to quadratic-linear systems.</p> <ul style="list-style-type: none"> <li>● I can find the solutions to linear systems algebraically and graphically.</li> <li>● I can use a variety of tools to solve quadratic-linear systems of equations.</li> <li>● I can interpret the points between lines and parabolas that indicate solution sets for a system of linear and quadratic equations.</li> </ul>
<p><b>Cluster: Create equations that describe numbers or relationships.</b></p>		
<p><b>KY.HS.A.14</b> Create a system of equations or inequalities to represent constraints within a modeling context. Interpret the solution(s) to the corresponding system as viable or nonviable options within the context. <b>MP.4, MP.5, KILP.3, KILP.9, KILP.10</b></p> <p><i>Supporting Standard(s):</i> <a href="#">KY.HS.A.12</a>, <a href="#">KY.HS.A.13</a>, &amp; <a href="#">KY.HS.A.20</a></p> <p><input type="checkbox"/> Conceptual    <input type="checkbox"/> Procedural    <input type="checkbox"/> Application</p>	<p>Students may be asked to find an optimal solution and the conditions under which the optimal solution would occur for a given real-world situation.</p> <p>Supplement with Linear Programming.</p>	<p>We are learning to interpret the solutions to the corresponding system.</p> <ul style="list-style-type: none"> <li>● I can create a system of equations or inequalities.</li> <li>● I can understand that a system of equations or inequalities is a set of mathematical expressions representing relationships between variables.</li> <li>● I can solve and interpret solutions to a system of equations or inequalities in context as viable or nonviable options within the context of the problem</li> </ul>
<p><b>Supporting Standards</b></p>		
<p><b>KY.HS.A.3</b> Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★</p> <p>a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient and constant term.</p> <p>b. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>c. Use the properties of exponents to rewrite exponential expressions.</p> <p>d. (+) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. <b>MP.5, MP.7</b></p> <p><b>KY.HS.A.7</b> Identify roots of polynomials when suitable factorizations are available. Know these roots become the zeros (x-intercepts) for the corresponding polynomial function. <b>MP.2, MP.5, MP.7</b></p> <p><b>KY.HS.A.12</b> Create equations and inequalities in one variable and use them to solve problems. <b>MP.1, MP.4</b></p> <p><b>KY.HS.A.13</b> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels</p>		

and scales. **MP.2, MP.5**

**KY.HS.A.17** Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise. a. Solve rational equations written as proportions in one variable. b. Solve radical equations in one variable. **MP.3, MP.5, MP.7**

**KY.HS.A.20** Solve systems of linear equations in two variables.

- a. Understand a system of two equations in two variables has the same solution as a new system formed by replacing one of the original equations with an equivalent equation.
- b. Solve systems of linear equations with graphs, substitution and elimination, focusing on pairs of linear equations in two variables. **MP.3, MP.6**

**KY.HS.A.22 (+)** Use matrices to solve a system of equations.

- a. Represent a system of linear equations as a single matrix equation in a vector variable.
- b. Find the inverse of a matrix if it exists.
- c. Use matrices to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater). **MP.4, MP.7**

**KY.HS.F.4** Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). ★

- f. (+) Graph piecewise functions, including step functions.

**KY.HS.N.7** Understanding properties of complex numbers.

- a. Know there is a complex number  $i$  such that  $i^2 = -1$  and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- b. Use the relation  $i^2 = -1$  and the commutative, associative and distributive properties to add, subtract and multiply complex numbers.
- c. (+) Find the conjugate of a complex number and use it to find the quotient of complex numbers.

**MP.7, MP.8**

**KY.HS.N.14** Use matrices to represent and manipulate data. **MP.4, MP.5**

**KY.HS.N.15** Perform operations with matrices.

- a. Add, subtract and multiply matrices of appropriate dimensions.
- b. Multiply matrices by scalars to produce new matrices. **MP.7, MP.8**

### Essential Vocabulary

average rate of change  
common difference  
maximum/minimum  
interval notation  
set builder notation  
sigma notation

explicit definition  
recursive definition  
sequence  
series  
dimensions  
solution of a system of linear equations

vertex  
parabola  
quadratic function  
vertex form of a quadratic function  
standard form of a quadratic function  
zero of a function

solution  
 system of equations/inequalities  
 piecewise-defined function  
 step function  
 arithmetic sequence/series  
 arithmetic sequence  
 arithmetic series  
 common difference

system of linear equations  
 system of linear inequalities  
 optimal solution  
 reduced row echelon form  
 complex number  
 imaginary number  
 imaginary unit  $i$   
 complex conjugates

zero product property  
 perfect square trinomial  
 completing the square  
 radicand  
 discriminant  
 Quadratic Formula  
 radicand  
 perfect square trinomial  
 discriminant

**Standards Benchmark Assessment #1**

November 2024

**Anchor Resources**

enVision Topic 1 - Linear Functions and Systems	enVision Topic 2 - Quadratic Functions and Equations
<p><b>MILC Resources</b> - <a href="#">Linear Functions and Systems Resources</a></p> <p><b>3 ACT Math Task (one per unit)</b> <i>enVision “Current Events” and Ior Falling glow sticks</i></p> <p><b>STEM Task - Fuel Efficiency for Cars (2 minute NBC video)</b>                      Students can research other types of vehicles i.e.minivans or trucks.</p> <p><b>Desmos</b> - <a href="#">Desmos: Point Collector Lines</a> (with lesson 1-6)</p>	<p><b>MILC Resources</b> - <a href="#">Quadratic Functions and Equations Resources</a></p> <p><b>3 ACT Math Task (one per unit)</b> <i>enVision “Swift Kick”</i></p> <p><b>STEM Task - How the design of a baseball park influences the number and frequency of home runs (2 minute NBC video)</b></p> <p><b>Desmos</b> - <a href="#">Desmos Intro to Vertex Form</a> (with lesson 2-1)</p>

**Formative Assessment Lesson (FAL)** - (one per semester as a minimum) examples to include:

- FAL - [Maximizing Profits: Selling Boomerangs](#) (after 1-6)
- FAL - [Solving Quadratic Equations](#) (after 2-6)
- FAL - [Solving Linear Equations in Two Variables](#) (after 1-6)
- FAL - [Representing Quadratic Functions Graphically](#) (after 2-3)

\*Disclaimer: Success Criteria is the evidence students must produce to demonstrate learning. This example is not comprehensive.

\*\* Mathematical Practices (A.MP.1- 8) should be evidenced at some point throughout each unit, depending on the explored tasks. It is important to note that MP. 2 should support learning in every lesson.

\*\*\* Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to *all* standards in that group.