





**EXAMPLE 3** Try It! Use Expected Values to Evaluate Strategies

3. The insurance company in Example 3 also offers optional safety glass coverage. Annual windshield repair statistics are: 50% no repairs, 30% minor repairs (\$50), and 20% full replacement (\$300). Should the owner opt for safety glass coverage? Which option has the lower expected cost?

Option	Premium (\$)	Deductible (\$)
Deductible	50	200
No Deductible	100	0

**EXAMPLE 4** Try It! Use Binomial Probability to Find Expected Value

4. A carnival game has 4 orange lights and 1 green light that flash rapidly one at a time in a random order. When a player pushes a button, the game stops, leaving one light on. If the light is green, the player wins a prize. Copy and complete the table, then determine the number of prizes that a player can expect to win if the game is played 4 times.

Number of Green Lights (wins)	Probability
0	${}_4C_0(0.2)^0(0.8)^4 = \square$
1	${}_4C_1(0.2)^1(0.8)^3 = \square$
2	${}_4C_2(0.2)^2(0.8)^2 = \square$
3	${}_4C_3(0.2)^3(0.8)^1 = \square$
4	${}_4C_4(0.2)^4(0.8)^0 = \square$

**HABITS OF MIND**

**Generalize** When do you add expected values and when do you compare individual expected values? Explain.



## Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What does expected value tell about situations involving probability?

2. **Error Analysis** Benjamin is finding the expected value of the number of heads when tossing a fair coin 10 times. What is Benjamin's error?

Toss a coin 10 times   
 $E = 50\%$

3. **Construct Arguments** A carnival game costs \$1 to play. The expected payout for each play of this game is \$1.12. Should the carnival operators modify the game in way? Explain.

4. **Reason** The students in Ms. Kahn's class are raising money to help earthquake victims. They expect to raise \$0.52 for each raffle ticket they sell. If each raffle ticket is sold for \$2, what can you conclude?

5. **Vocabulary** A spinner is divided into 6 equal-sized sectors, numbered 1, 1, 1, 4, 7, and 10. Is the expected value of a spin the same as the mean of the numbers? Explain.

## Do You KNOW HOW?

6. What is the expected value when rolling a standard number cube?

7. What is the expected value when rolling two standard number cubes?

8. A travel website reports that in a particular European city, the probability of rain on any day in April is 40%. What is the expected number of rainy days in this city during the month of April?

9. You buy an airplane ticket for \$900. You discover that if you cancel or rebook your vacation flight to Europe, you will be charged an extra \$300. There is a 20% chance that you will have to rebook your flight.

a. What is the expected value of the cost of the ticket?

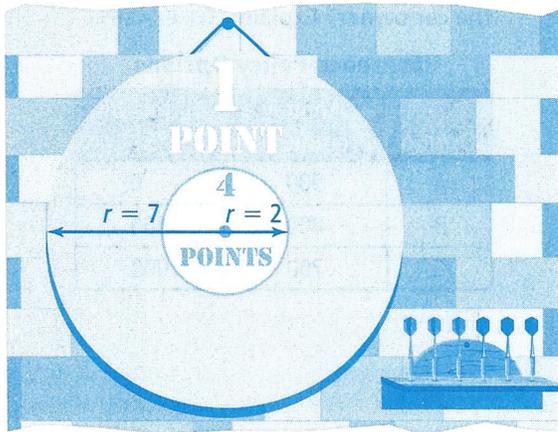
b. Is the expected value the amount you will pay to book the ticket whether or not you have to rebook? Explain.

10. A child-care service charges families an hourly rate based upon the age of the child. Their hourly rate per child is \$20 per hour for infants less than 1 year old, \$18 for toddlers 1 to 3 years old, \$15 per hour children 3 or more years old. The ratio of infants : toddlers : 3+ years is 2 : 3 : 5. What is the expected charge per child per hour?

## PRACTICE & PROBLEM SOLVING

### UNDERSTAND

11. **Error Analysis** For the dartboard shown, Deshawn calculated the expected number of points per dart. Explain Deshawn's error. What is the correct expected value?



$$\begin{aligned} \text{Expected value} &= \frac{2}{7}(4) + \frac{5}{7}(1) \\ &= \frac{8}{7} + \frac{5}{7} \\ &= \frac{13}{7} \approx 1.86 \end{aligned}$$

X

12. **Reason** A nonrefundable plane ticket costs \$600, while a refundable ticket costs \$900. A traveler estimates there is a 20% chance he will have to cancel his upcoming trip. Should the traveler purchase a refundable or nonrefundable ticket? Explain.

13. **Construct Arguments** A consumer determines that her expected cost for Option B is \$528 per year.

Option	Annual Premium	Deductible
A	\$600	\$0
B	\$500	\$1,000

- a. Why might this consumer select the policy with the \$1000 deductible?
- b. Why might this consumer select the policy with no deductible?

14. **Mathematical Connections** How is expected value related to the mean?

## PRACTICE & PROBLEM SOLVING

### PRACTICE

A farmer estimates her hens will produce 3,000 dozen more eggs this year than last year. She estimates the probability of her net profit or loss on each dozen eggs based on her costs.

SEE EXAMPLE 1

Egg production last year: 12,000 dozen

Estimated Net Profit per Dozen Eggs						
Net profit (¢ per doz.)	8	6	4	2	0	-2
Probability	0.1	0.4	0.2	0.1	0.1	0.1

15. What is her expected profit per dozen eggs?

16. What is her expected profit on the total egg production?

17. An electronics store offers students a discount of 10% on purchases of computers. They estimate that  $\frac{1}{16}$  of computer sales are to students. The average sale per customer is \$498 and the store's profit is \$80 before the discount. What is the expected profit on the sale of a computer? SEE EXAMPLE 2

18. An insurance company offers three policy options. The probability a car will be damaged in a given year is 5%, and if a car is damaged, the cost of the repairs will be \$1000. Which option has the least expected annual cost for the car owner? Explain. SEE EXAMPLE 3

#### Insurance Policy Options

Option	Annual Premium (\$)	Deductible (\$)
A	900	0
B	800	400
C	700	1000

On a tropical island, the probability of sunny weather is 90% each day. SEE EXAMPLE 4

19. What is the expected number of sunny days in a non-leap year?

20. What is the expected number of sunny days during the month of June?

**PRACTICE & PROBLEM SOLVING****APPLY**

- 21. Model With Mathematics** A solar panel company has found that about 1% of its panels are defective. The company's cost to replace each defective panel is \$600. A consultant recommends changes to the manufacturing process that will cost \$200,000 and reduce the defective rate to 0.2%. The company estimates that it will sell 30,000 panels next year and that sales will increase by 5,000 panels per year for the next 10 years. Should the company follow the consultant's recommendation? Explain.
- 22. Reason** A student tosses a coin 4 times and the results are heads, tails, heads, and heads. The student concludes that the expected number of heads for 100 tosses is 75. How did the student find this number? Do you agree with the student's reasoning? Explain.
- 23. Higher Order Thinking** Your family is going to buy a new TV set for \$599. You find out that the probability that the TV set will need to be serviced in the second year is 0.05 and the probability that the TV set will need to be serviced in the third year is 0.08. A 2-year warranty costs \$55, and a 3-year warranty costs \$80. The average cost of repairing the TV set is \$278. What would you advise your family to do, get a 2-year extended warranty, a 3-year extended warranty or not to get any extended warranty? Explain your reasoning.
- 24. Make Sense and Persevere** A company makes tablets that are guaranteed for one year. On average, one out of every 200 tablets needs to be repaired or replaced within the first year. If a tablet needs to be repaired, the company loses an average of \$140. If the company sells 2,600,000 of the tablets in a year, what is their net profit on the sale of the tablets in that year?



**ASSESSMENT PRACTICE**

25. A commuter recorded data on the arrival time of his morning train each weekday for 5 weeks. According to the data, he should expect the train to be 1.16 minutes late on any given day. What are the missing values in the commuter's table?

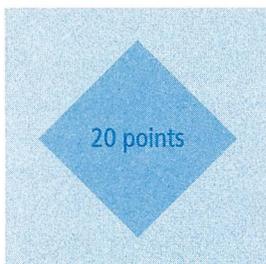
**Arrival Time for Train**

Minutes late	0	1	2	3	4	5
Number of days		5	1		1	3

26. **SAT/ACT** What is the expected total for 20 spins?
- (A) 100
  - (B) 105
  - (C) 110
  - (D) 115
  - (E) 120



27. **Performance Task** A toy company is designing a children's game in which players toss chips onto a board. The square board will contain a smaller square at its center.



**Part A** Write design instructions for the board so that a chip tossed randomly onto the board is 8 times more likely to land in the outer region than in the inner square. Explain your reasoning.

**Part B** Assign a whole number of points to the outer region so that the expected score on a single toss is as close as possible to 5. Explain your reasoning.

**Part C** If the area of the inner square is doubled and the overall size of the board remains the same, how does the expected score change? Is it also doubled? Explain.