

EXPLORE & REASON

Holly, Tia, Kenji, and Nate are eligible to be officers of the Honor Society. Two of the four students will be chosen at random as president and vice-president. The table summarizes the possible outcomes.

Honor Society Officers

		Vice-President			
		Holly	Tia	Kenji	Nate
President	Holly	–	HT	HK	HN
	Tia	TH	–	TK	TN
	Kenji	KH	KT	–	KN
	Nate	NH	NT	NK	–

- A. Holly wants to be an officer with her best friend Tia. How many outcomes make up this event?
- B. How many outcomes show Holly as president and Tia as vice-president?
- C. **Generalize** How many outcomes have only one of them as an officer? Explain.

HABITS OF MIND

Make Sense and Persevere How could you use the table to calculate the probability that both Holly and Tia will be officers?

EXAMPLE 1  **Try It! Use the Fundamental Counting Principle**

1. The car that Ms. Garcia is buying comes with a choice of 3 trim lines (standard, sport, or luxury), 2 types of transmission (automatic or manual), and 8 colors. How many different option packages does Ms. Garcia have to choose from? Explain.

EXAMPLE 2  **Try It! Find the Number of Permutations**

2. How many possibilities are there for each playlist?
 - a. Gabriela's 4 favorite songs
 - b. 5 of the 10 most popular songs

HABITS OF MIND

Communicate Precisely Explain how the Fundamental Counting principle can be used to find the number of ways to arrange 5 different colored beads on a string.

**EXAMPLE 3**  **Try It! Find the Number of Combinations**

3. How many ways can a camper choose 5 activities from the 10 available activities at the summer camp?

EXAMPLE 4  **Try It! Use Permutations and Combinations to Find Probabilities**

4. Using the data from Example 4, what is the probability that the 5 students' names end with a vowel?

HABITS OF MIND

Look for Relationships What is the relationship between ${}_{10}P_5$ and ${}_{10}C_5$? Is the number of permutations always greater than the number of combinations?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are permutations and combinations useful when finding probabilities?

2. **Use Structure** How is the formula for combinations related to the formula for a permutations?

3. **Vocabulary** Why is it important to distinguish between a *permutation* and a *combination* when counting possible outcomes?

4. **Look for Relationships** How is ${}_9C_2$ related to ${}_9C_7$? Explain. How can you generalize this observation for any values of n and r ?

5. **Error Analysis** Explain Beth's error.

$$\frac{{}_3P_3}{{}_5P_3} = \frac{3!}{5!} = \frac{3!}{5!2!} = \frac{1}{40}$$

X

6. **Construct Arguments** A company wants to form a committee of 4 people from its 12 employees. How can you use combinations to find the probability that the 4 people newest to the company will be selected?

Do You KNOW HOW?

Do the possible outcomes represent permutations or combinations?

7. Jennifer will invite 3 of her 10 friends to a concert.

8. Jennifer must decide how she and her 3 friends will sit at the concert.

Find the number of permutations.

9. How many ways can 12 runners in a race finish first, second, and third?

Find the number of combinations.

10. In how many ways can 11 contestants for an award be narrowed down to 3 finalists?

11. How many different ways can a 4-person team can be chosen from a group of 8 people?

Students will be chosen at random for school spirit awards. There are 6 athletes and 8 non-athletes who are eligible for 2 possible prizes. What is each probability?

12. P (both prizes are awarded to athletes)

13. P (both prizes are awarded to non-athletes)

14. P (no prize is awarded to an athlete)

15. P (no prize is awarded to a non-athlete)

16. Explain how Exercises 12 and 13 are similar to Exercises 14 and 15.

PRACTICE & PROBLEM SOLVING

UNDERSTAND

17. **Use Structure** Dwayne bought a new bike lock, and the lock came with instructions to choose 3 out of 30 numbers on a circular dial to keep his bike secure. The numbers cannot be repeated. How many possible arrangements can Dwayne choose for his lock? Do the arrangements represent permutations or combinations? Explain.

18. **Construct Arguments** Sage volunteers to read and play with sick children in a hospital. She selects some erasers at random from a bag to use as prizes. There are 8 alien erasers and 10 flying saucer erasers.

- How many groups of 6 erasers can be formed from the 18 erasers? Explain.
- In how many ways can 3 aliens be selected? Explain.
- In how many ways can 3 aliens and 3 flying saucers be selected? Explain.
- What is the probability that 3 aliens and 3 flying saucers will be selected? Explain.

19. **Error Analysis** There are 6 tiles numbered 1 to 6 in a box. Two tiles are selected at random without replacement to form a 2-digit number. Jeffrey found the probability that the number selected is 16. Explain his error.

The number of ways to select 1 and 6 is given by ${}_6C_2 = 15$

$$P(16) = \frac{1}{{}_6C_2} = \frac{1}{15} \quad \mathbf{X}$$

20. **Mathematical Connections** How many lines are determined by the points, P , Q , R , and S ? Explain.



21. **Higher Order Thinking** There are $11!$ different ways for a group of people to sit around a circular table. How many people are in the group? Explain.

PRACTICE & PROBLEM SOLVING

PRACTICE

For Exercises 22–27, state if the possible arrangements represent permutations or combinations, then state the number of possible arrangements. SEE EXAMPLES 1, 2, AND 3

- 22. A student chooses at random 4 books from a reading list of 11 books.

- 23. At the end of a season, 10 soccer teams are ranked by the state.

- 24. A committee of 5 people is being selected from a group of 9 to choose the food for a sport's banquet.

- 25. Hugo displays his 8 model planes in a single row.

- 26. A class president, secretary, and treasurer are chosen from 12 students running for office.

- 27. A food truck has a lunch special on tacos. Customers choose a shell, three toppings, and two sides for one price.



- 28. There are 4 comedians and 5 musicians performing in a variety show. The order in which the performers are chosen is random. SEE EXAMPLE 4
 - a. What is the probability that the first 3 performers are comedians?

 - b. What is the probability that the first two performers are a comedian followed by a musician?

- 29. A jewelry maker chooses three beads at random from a bag with 10 beads labeled A, B, C, D, E, F, G, H, I, and J. SEE EXAMPLES 2, 3, AND 4
 - a. How can you use permutations or combinations to find $P(\text{selected beads spell the initials DEB})$? What is the probability?

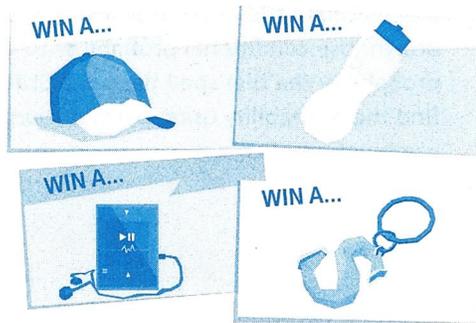
 - b. How can you use permutations or combinations to find $P(\text{selected beads are all vowels})$? What is the probability?

PRACTICE & PROBLEM SOLVING

APPLY

30. **Make Sense and Persevere** Amaya's wallet contains three \$1 bills, two \$5 bills, and three \$10 bills. If she pulls 2 bills without looking, what is the probability that she draws a \$1-bill and a \$10-bill? Explain.

31. **Model with Mathematics** Raul's favorite restaurant is running a prize game. Five of each of the winning tickets shown are available, and a customer must collect three winning tickets to receive the prize. What is the probability Raul will receive the prize for the baseball cap with his first 3 tickets?

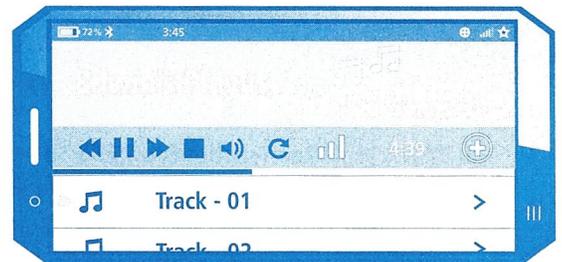


32. **Look for Relationships** Smart Phones, Inc. chooses a 5-digit security code at random from the digits 0–9.

a. Suppose the digits cannot be repeated. What is the probability that the security code is 30429? Explain.

b. Suppose the digits can be repeated. What is the probability that the security code is 30429? Explain.

33. **Make Sense and Persevere** Edwin randomly plays 6 different songs from his playlist.



a. What is the probability that Edwin hears his 6 favorite songs?

b. What is the probability that he hears the songs in order from his most favorite to his sixth most favorite?

ASSESSMENT PRACTICE

34. Consider an arrangement of 8 items taken 3 at a time in which order is not important. Does each expression give the correct number of arrangements? Select Yes or No.

	Yes	No
8P_3	<input type="checkbox"/>	<input type="checkbox"/>
8C_3	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{{}^8P_3}{3!}$	<input type="checkbox"/>	<input type="checkbox"/>
$8! \cdot 3!$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{8!}{3!}$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{8!}{5!}$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{8!}{3!5!}$	<input type="checkbox"/>	<input type="checkbox"/>
$8 \cdot 7$	<input type="checkbox"/>	<input type="checkbox"/>

35. **SAT/ACT** Fifteen students enter a Safety Week poster contest in which prizes will be awarded for first through fourth place. In how many ways could the prizes be given out?
- (A) 4
 (B) 60
 (C) 1,365
 (D) 32,760
 (E) 50,625

36. **Performance Task** Use the word shown on the tiles below to find each probability.



Part A Two tiles are chosen at random without replacement. Use conditional probability to find the probability that both letters are vowels. Then find the probability using permutations or combinations. Explain.

Part B Four of the tiles are chosen at random and placed in the order in which they are drawn. Use conditional probability to find the probability the tiles spell the word SURF. Then find the probability using permutations or combinations. Explain.