6-6 Exponential and Logarithmic Equations

MODEL & DISCUSS

A store introduces two new models of fitness trackers to its product line. A glance at the data is enough to see that sales of both types of fitness trackers are increasing. Unfortunately, the store has limited space for the merchandise. The manager decides that the store will sell both models until sales of TrackSmart exceed those of FitTracker.

	FitTracker *Number Sold*	**TrackSmart** *Number Sold*
Week 4	228	130
Week 3	112	44
Week 2	54	17
Week 1	28	5

A. **Model With Mathematics** Find an equation of an exponential that models the sales for each fitness tracker. Describe your method.

B. Based on the equations that you wrote, determine when the store will stop selling FitTracker.

HABITS OF MIND

Look for Relationships How do you know that the sales data is modeled by an exponential function?

Notes

Assess

EXAMPLE 1

Try It! Solve Exponential Equations Using a Common Base

1. Solve each equation using a common base.

a. $25^{3x} = 125^{x+2}$

b. $0.001 = 10^{6x}$

EXAMPLE 2

Try It! Rewrite Exponential Equations Using Logarithms

2. Rewrite the equation $5^x = 12$ using logarithms.

HABITS OF MIND

Communicate Precisely In order to set the exponents of two exponential expressions equal to each other, what must be true about the exponential expressions?

Notes

Assess

EXAMPLE 3 Try It! Solve Exponential Equations Using Logarithms

3. What is the solution to $2^{3x} = 7^{x+1}$?

EXAMPLE 4 Try It! Use an Exponential Model

4. About how many minutes does it take the fire to spread to cover 100 acres?

HABITS OF MIND

Use Structure Why is it useful to use logarithms to solve an exponential equation?

EXAMPLE 5 Try It! Solve Logarithmic Equations

5. Solve each equation.

a. $\log_5 (x^2 - 45) = \log_5 (4x)$

b. $\ln (-4x - 1) = \ln (4x^2)$

EXAMPLE 6 Try It! Solve Logarithmic and Exponential Equations by Graphing

6. Solve each equation by graphing. Round to the nearest thousandth.

a. $3(2)^{x+2} - 1 = 3 - x$

b. $\ln (3x - 1) = x - 5$

HABITS OF MIND

Generalize Summarize the procedure for solving a logarithmic equation.

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How do properties of exponents and logarithms help you solve equations?

2. **Vocabulary** Jordan claims that $x^2 + 3 = 12$ is an exponential equation. Is Jordan correct? Explain your thinking.

3. **Communicate Precisely** How can properties of logarithms help to solve an equation such as $\log_6 (8x - 2)^3 = 12$?

Do You KNOW HOW?

Solve. Round to the nearest hundredth, if necessary. List any extraneous solutions.

4. $16^{3x} = 256^{x+1}$

5. $6^{x+2} = 4^x$

6. $\log_5 (x^2 - 44) = \log_5 (7x)$

7. $\log_2 (3x - 2) = 4$

8. $4^{2x} = 9^{x-1}$

9. A rabbit farm had 200 rabbits in 2015. The number of rabbits increases by 30% every year. How many rabbits are on the farm in 2031?

PRACTICE & PROBLEM SOLVING

UNDERSTAND

10. **Use Structure** Would you use the natural log or the common log when solving the equation $10^{x+2} = 78$? Is it possible to use either the natural log or common log? Explain.

11. **Make Sense and Persevere** Explain why logarithms are necessary to solve the equation $3^{x+2} = 8$, but are not necessary to solve the equation $3^{x+2} = 27^{4x}$.

12. **Reason** Tristen solved the equation $\log_3 (x + 1) - \log_3 (x - 6) = \log_3 (2x + 2)$. Justify each step of solving the equation in Tristen's work. Are both numbers solutions to the equation? Explain.

$$\log_3(x + 1) - \log_3(x - 6) = \log_3(2x + 2)$$
$$\log_3(x + 1) = \log_3(2x + 2) + \log_3(x - 6)$$
$$\log_3(x + 1) = \log_3(2x + 2)(x - 6)$$
$$(x + 1) = (2x + 2)(x - 6)$$
$$x + 1 = 2x^2 - 10x - 12$$
$$0 = 2x^2 - 11x - 13$$
$$x = 6.5 \text{ or } x = -1$$

13. **Error Analysis** The number of milligrams of medicine in a person's system after t hours is given by the function $A = 20e^{-0.40t}$. Thomas sets $A = 0$ to find the number of hours it takes for all of the medicine to be removed from a person's system. What mistake did Thomas make? Explain.

14. **Mathematical Connections** Explain the importance of the Power Property of Logarithms when solving exponential equations.

15. **Error Analysis** Find the student error in the solution of the logarithmic equation.

$$\log (x + 3) + \log x = 1$$
$$\log x (x + 3) = 1$$
$$x(x + 3) = 10^1$$
$$x^2 + 3x - 10 = 0$$
$$(x - 2)(x + 5) = 0$$
$$x = 2, -5$$

X

PRACTICE & PROBLEM SOLVING

PRACTICE

Find all solutions of the equation. Round answers to the nearest ten-thousandth. SEE EXAMPLE 1

16. $3^{2-3x} = 3^{5x-6}$

17. $7^{3x} = 54$

18. $25^{x^2} = 125^{x+3}$

19. $4^{3x-1} = \left(\frac{1}{2}\right)^{x+5}$

20. $4^{2x+1} = 4^{3x-5}$

21. $6^{x-2} = 216$

Find all solutions of the equation. Round answers to the nearest ten-thousandth. SEE EXAMPLES 2 AND 3

22. $2^{3x-2} = 5$

23. $4 + 5^{6-x} = 15$

24. $6^{3x+1} = 9^x$

25. $-3 = \left(\frac{1}{2}\right)^x - 12$

26. $3^{2x-3} = 4^x$

27. $4^{x+2} = 8^{x-1}$

28. Dale has \$1,000 to invest. He has a goal to have \$2,500 in this investment in 10 years. At what annual rate compounded continuously will Dale reach his goal? Round to the nearest hundredth. SEE EXAMPLE 4

Find all solutions of the equation. Round answers to the nearest thousandth. SEE EXAMPLE 5

29. $\log_2 (4x + 5) = \log_2 x^2$

30. $2\ln (3x - 2) = \ln (5x + 6)$

31. $\log_4 (x^2 - 2x) = \log_4 (3x + 8)$

32. $\ln (5x - 2) = \ln (x - 1)$

33. $\ln (2x^2 + 5x) = \ln (2x + 7)$

34. $2\log (x + 1) = \log (x + 1)$

35. $\log_2 x + \log_2 (x - 3) = 2$

36. $\log_2 (3x - 2) = \log_2 (x - 1) + 4$

37. $\log_6 (x^2 - 2x) = \log_6 (2x - 3) + \log_6 (x + 1)$

Solve by graphing. Round answers to the nearest thousandth. SEE EXAMPLE 6

38. $\log(5x - 3)^2 = x - 4$

39. $\ln(2x) = 3x - 5$

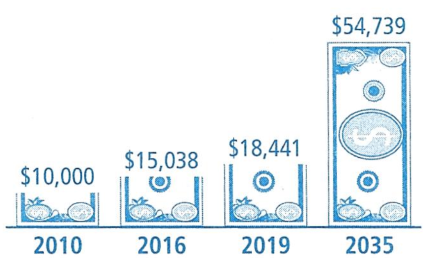
40. $\log(4x) = x + \log x$

PRACTICE & PROBLEM SOLVING

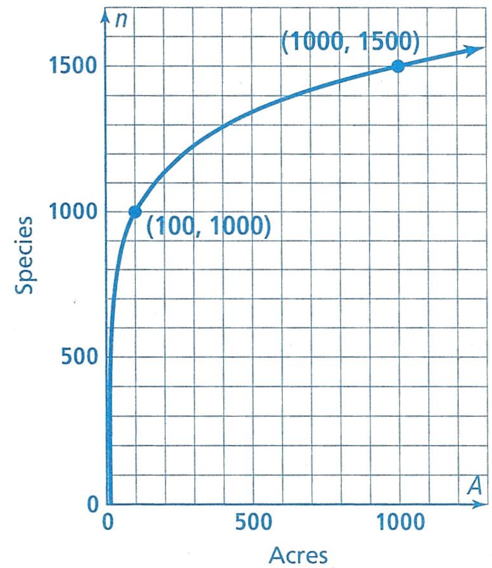
APPLY

41. Model With Mathematics The population of a city is modeled by the function $P = 250{,}000e^{0.013t}$, where t is the number of years since 2000. In what year, to the nearest year, will the population reach 450,000?

42. Use Structure Felix invested \$10,000 into a retirement account in 2010. He then projected the amount of money that would be in the account for several years assuming that interest would compound continuously at an annual rate. Later, when he looked back the data, he could not recall the annual rate that he used for the projections. Use the data below to determine the annual rate.



43. Higher Order Thinking A biologist is using the logarithmic model $n = k\log(A)$ to determine the number of a species n, that can live on a land mass of area A. The constant k varies according to the species.



a. Use the graph to determine the constant k for the species that the scientist is studying.

b. Determine the land mass in acres that is needed to support 3,000 of the species.

ASSESSMENT PRACTICE

44. Which of the following have the same solution? Select all that apply.

Ⓐ $\log_8 (x^2 - 15) = \log_8 (2x)$

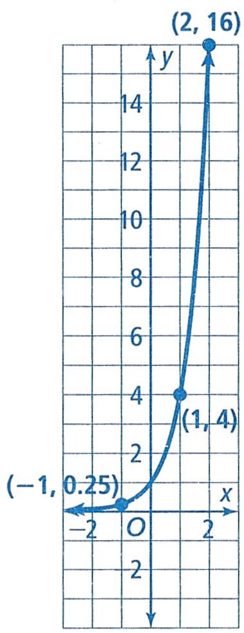
Ⓑ $\ln (12x + 2) = \ln (2x - 3)$

Ⓒ $\log_2 x + \log_2 (x + 4) = 5$

Ⓓ $\log_3 (15x + 6)^2 = 8$

Ⓔ $\log_4 (3x - 5) = 2$

45. SAT/ACT The graph shows the function $y = 4^x$. Determine when the function shown in the graph is greater than the function $y = 2^{3x-1}$.



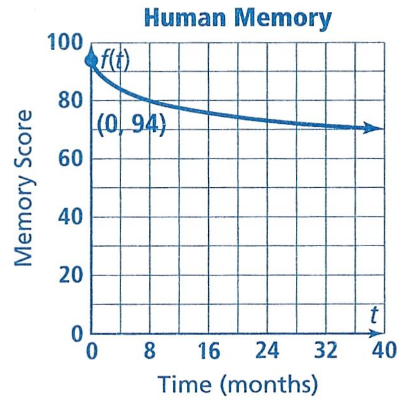
Ⓐ $x > 1$

Ⓑ $x < 1$

Ⓒ $x > -1$

Ⓓ $x < -1$

46. Performance Task A professor conducted an experiment to find the relationship between time and memory. The professor determined the model $f(t) = t_0 - 15 \log (t + 1.1)$ gives the memory score after t months when a student had an initial memory score of t_0.



Part A Write a model for a student with the given initial memory score.

Part B After about how many years will the student have a memory score of 65?