Activity

6-1 Key Features of Exponential Functions

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EXPLORE & REASON

Margaret investigates three functions: $y = 3x$, $y = x^3$, and $y = 3^x$. She is interested in the differences and ratios between consecutive y-values. Here is the table she started for $y = 3x$.

Investigating $y = 3x$			
x	y	Difference between y-values	Ratio between y-values
1	3		
2	6	$6 - 3 = 3$	$\frac{6}{3} = 2$
3	9	$9 - 6 = 3$	$\frac{9}{6} = 1.5$
4	12	$12 - 9 = 3$	$\frac{12}{9} \approx 1.33$

A. Create tables like Margaret's for all three functions and fill in more rows.

B. Which functions have a constant difference between consecutive y-values? Constant ratio?

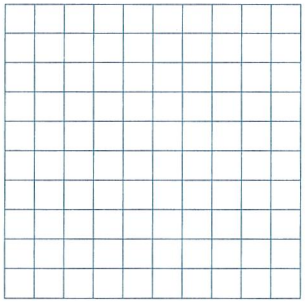
C. Use Structure Which of these three functions will have y-values that increase the fastest as x increases? Why?

HABITS OF MIND

Generalize Let b represent a whole number. For $b > 1$, which function do you think will increase at a faster rate as x increases, $f(x) = b^x$ or $g(x) = x^b$? Explain.

EXAMPLE 1 Try It! Identify Key Features of Exponential Functions

1. Graph $f(x) = 4(0.5)^x$. What are the domain, range, intercepts, asymptote, and the end behavior for this function?



EXAMPLE 2 Try It! Graph Transformations of Exponential Functions

2. How do the asymptote and intercept of the given function compare to the asymptote and intercept of the function $f(x) = 5^x$?

   a. $g(x) = 5^{x+3}$

   b. $h(x) = 5^{-x}$

HABITS OF MIND

Reason What kinds of transformations will affect the asymptote or the intercept(s) of an exponential function? Explain.

EXAMPLE 3 Try It! Model with Exponential Functions

3. A factory purchased a 3D Printer on January 2, 2010. The value of the printer is modeled by the function $f(x) = 30(0.93)^x$, where x is the number of years since 2010.
   a. What is the value of the printer after 10 years?

   b. Does the printer lose more of its value in the first 10 years or in the second?

EXAMPLE 4 Try It! Interpret an Exponential Function

4. Two-hundred twenty hawks were released into a region in 2016. The function $f(x) = 220(1.05)^x$ can be used to model the number of red-tailed hawks in the region x years after 2016.
   a. Is the population increasing or decreasing? Explain.

   b. In what year will the number of hawks reach 280?

HABITS OF MIND

Use Structure How can you determine the growth or decay factor by looking at an exponential function? The growth or decay rate?

EXAMPLE 5 Try It! Compare Two Exponential Functions

5. In Example 5, will the value of the painting ever surpass the value of the sculpture according to the models? Explain.

HABITS OF MIND

Reason For two functions $f(x) = b^x$ and $g(x) = b^{x+n}$, where $n > 0$, is it possible that the two graphs will intersect? Explain.

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How do graphs and equations reveal key features of exponential growth and decay functions?

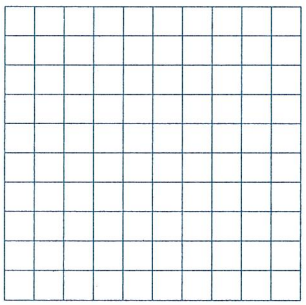
2. **Vocabulary** How do *exponential functions* differ from polynomial and rational functions?

3. Error Analysis Charles claimed the function $f(x) = \left(\frac{3}{2}\right)^x$ represents exponential decay. Explain the error Charles made.

4. Communicate Precisely How are exponential growth functions similar to exponential decay functions? How are they different?

Do You KNOW HOW?

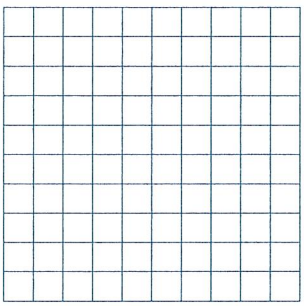
5. Graph the function $f(x) = 4 \times 3^x$. Identify the domain, range, intercept, asymptote, and describe the end behavior.



6. The exponential function $f(x) = 2500(0.4)^x$ models the amount of money in Zachary's savings account over the last 10 years. Is Zachary's account balance increasing or decreasing? Write the base in terms of the rate of growth or decay.

7. Describe how the graph of $g(x) = 4(0.5)^{x-3}$ compares to the graph of $f(x) = 4(0.5)^x$.

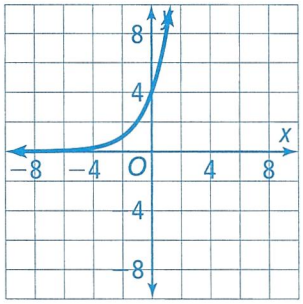
8. Two trucks were purchased by a landscaping company in 2016. Their values are modeled by the functions $f(x) = 35(0.85)^x$ and $g(x) = 46(0.75)^x$ where x is the number of years since 2016. Which function models the truck that is worth the most after 5 years? Explain.



PRACTICE & PROBLEM SOLVING

UNDERSTAND

9. Use Structure What value of a completes the equation $y = a \cdot 2^x$ for the exponential growth function shown below?



10. Make Sense and Persevere Cindy found a collection of baseball cards in her attic worth $8,000. The collection is estimated to increase in value by 1.5% per year. Write an exponential growth function and find the value of the collection after 7 years.

11. Error Analysis Describe and correct the error a student made in identifying the growth or decay factor for the function $y = 2.55(0.7)^x$.

Step 1 The base of the function is 0.7, so it represents exponential decay.

Step 2 The function in the form $y = a(1 - r)^x$ is $y = 2.55(1 - 0.7)^x$. ✗

Step 3 The decay factor is 0.3.

12. Reason In 2000, the population of St. Louis was 346,904, and it decreased to 319,257 in 2010. If this population decrease were modeled by an exponential decay function, what value would represent the y-intercept? Explain your reasoning.

13. Mathematical Connections Describe how the graph of $g(x) = 6 \cdot 2^{x+1} - 4$ compares to the graph of $f(x) = 6 \cdot 2^x$.

PRACTICE & PROBLEM SOLVING

PRACTICE

Identify the domain, range, intercept, and asymptote of each exponential function. Then describe the end behavior. SEE EXAMPLE 1

14. $f(x) = 5 \cdot 3^x$

15. $f(x) = 0.75\left(\frac{2}{3}\right)^x$

16. $f(x) = 4\left(\frac{1}{2}\right)^x$

17. $f(x) = 7 \cdot 2^x$

Determine whether each function represents exponential growth or decay. Write the base in terms of the rate of growth or decay, identify *r*, and interpret the rate of growth or decay. SEE EXAMPLES 3 AND 4

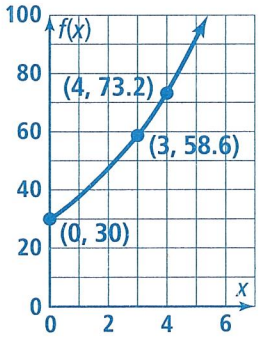
18. $y = 100 \cdot 2.5^x$

19. $f(x) = 10{,}200\left(\frac{3}{5}\right)^x$

20. $f(x) = 12{,}000\left(\frac{7}{10}\right)^x$

21. $y = 450 \cdot 2^x$

22. The function $f(x)$, shown in the graph, represents an exponential growth function. Compare the average rate of change of $f(x)$ to the average rate of change of the exponential growth function $g(x) = 25\ (1.4)^x$. Use the interval [0, 4]. SEE EXAMPLE 5



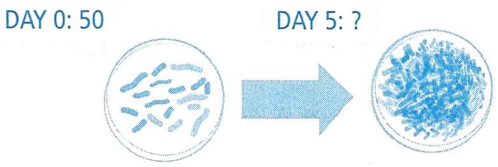
23. Write a function $g(x)$ that represents the exponential function $f(x) = 2^x$ after a vertical stretch of 6 and a reflection across the x-axis. Graph both functions. SEE EXAMPLE 2

24. The population of Medway, Ohio, was 4,007 in 2000. It is expected to decrease by about 0.36% per year. Write an exponential decay function and use it to approximate the population in 2020. SEE EXAMPLE 4

PRACTICE & PROBLEM SOLVING

APPLY

25. Model With Mathematics A colony of bacteria starts with 50 organisms and quadruples each day. Write an exponential function, $P(t)$, that represents the population of the bacteria after t days. Then find the number of bacteria that will be in the colony after 5 days.



26. Higher Order Thinking The number of teams y remaining in a single elimination tournament can be found using the exponential function $y = 128\left(\frac{1}{2}\right)^x$, where x is the number of rounds played in the tournament.

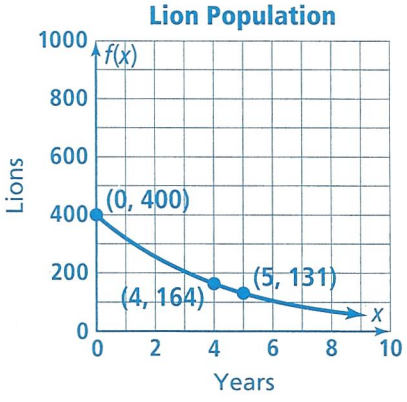
a. Determine whether the function represents exponential growth or decay. Explain.

b. What does 128 represent in the function?

c. What percent of the teams are eliminated after each round? Explain how you know.

d. Graph the function. What is a reasonable domain and range for the function? Explain.

27. Construct Arguments The function shown in the graph represents the number of lions in a region after x years, where the rate of decay is 20%. The number of zebras in that same region after x years can be modeled by the function $f(x) = 300(0.95)^x$. A representative for a conservationist group claims there will be fewer lions than zebras within 2 years. Is the representative correct? Justify your answer.

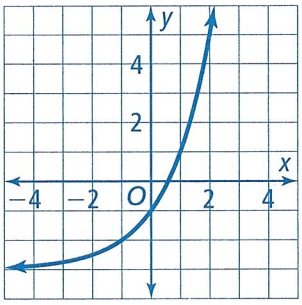


ASSESSMENT PRACTICE

28. The exponential function $g(x) = 3^{x-1} + 6$ is a transformation of the function $f(x) = 3^x$. Does each statement accurately describe how the graph of $g(x)$ compares to the graph of $f(x)$? Select yes or no.

	Yes	No
a. $g(x)$ is translated 6 units up.	❑	❑
b. $g(x)$ is translated 6 units down.	❑	❑
c. $g(x)$ is translated 6 units to the right.	❑	❑
d. $g(x)$ is translated 1 unit to the right.	❑	❑
e. $g(x)$ is translated 1 unit to the left.	❑	❑
f. The horizontal asymptote shifts 1 unit down.	❑	❑

29. SAT/ACT Which of the functions defined below could be the one shown in this graph?



Ⓐ $f(x) = 4(2)^{x-1} + 3$

Ⓑ $f(x) = 4(2)^{x+1} + 3$

Ⓒ $f(x) = 4(2)^{x-1} - 3$

Ⓓ $f(x) = 4(2)^{x+1} - 3$

30. Performance Task A radioactive isotope of the element osmium Os-182 has a half-life of 21.5 hours. This means that if there are 100 grams of Os-182 in a sample, after 21.5 hours there will only be 50 grams of that isotope remaining.

Part A Write an exponential decay function to model the amount of Os-182 in a sample over time. Use A_0 for the initial amount and A for the amount after time t in hours.

Part B Use your model to predict how long it would take a sample containing 500 g of Os-182 to decay to the point where it contained only 5 g of Os-182.