CRITIQUE & EXPLAIN

Nicky and Tavon used different methods to solve the equation $\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}$.

Nicky	Tavon
$\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}$	$\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}$
$\frac{1}{2}x = \frac{9}{10} - \frac{2}{5}$	$10\left(\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}\right)$
$\frac{1}{2}x = \frac{5}{10}$	$5x + 4 = 9$
$x = 1$	$5x = 5$
	$x = 1$
The solution is 1.	The solution is 1.

A. Explain the different strategies that Nicky and Tavon used and the advantages or disadvantages of each.

B. Did Nicky use a correct method to solve the equation? Did Tavon?

C. Use Structure Why might Tavon have chosen to multiply both sides of the equation by 10? Could he have used another number? Explain.

HABITS OF MIND

Reason If Tavon had multiplied both sides of the equation by 100, would his answer have been 10 times as much? Explain.

EXAMPLE 1 ☑ **Try It!** **Solve a Rational Equation**

1. What is the solution to each equation?

   a. $\dfrac{2}{x+5} = 4$

   b. $\dfrac{1}{x-7} = 2$

HABITS OF MIND

Critique Reasoning For part (b), Kaitlyn wrote $1 = 2x - 7$, then $8 = 2x$, and $4 = x$. Is she correct? Explain.

EXAMPLE 2 ☑ **Try It!** **Solve a Work-Rate Problem**

2. It takes 12 hours to fill a pool with two pipes, where the water in one pipe flows three times as fast as the other pipe. How long will it take the slower pipe to fill the pool by itself?

HABITS OF MIND

Reason In a Work-Rate problem, explain why you can't average the individual rates to determine how long it will take to complete a job together.

EXAMPLE 3   ☑ **Try It!** **Identify an Extraneous Solution**

3. What is the solution to the equation $\frac{1}{x+2} + \frac{1}{x-2} = \frac{4}{(x+2)(x-2)}$?

EXAMPLE 4   ☑ **Try It!** **Solve Problems With Extraneous Solutions**

4. What are the solutions to the following equations?

a. $x + \frac{6}{x-3} = \frac{2x}{x-3}$

b. $\frac{x^2}{x+5} = \frac{25}{x+5}$

HABITS OF MIND

Communicate Precisely What is an extraneous solution?

EXAMPLE 5   ☑ **Try It!** **Solve a Rate Problem**

5. Three people are planting tomatoes in a community garden. Marta takes 50 minutes to plant the garden alone, Benito takes x minutes and Tyler takes $x + 15$ minutes. If the three of them take 20 minutes to finish the garden, how long would it have taken Tyler alone?

HABITS OF MIND

Make Sense and Persevere What does the fraction $\frac{1}{50}$ mean with regards to Marta?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you solve rational equations and identify extraneous solutions?

2. **Vocabulary** Write your own example of a rational equation that, when solved, has at least one **extraneous solution**.

3. **Error Analysis** A student solved the rational equation as follows:

$$\frac{1}{2x} - \frac{2}{5x} = \frac{1}{10x} - 3; x = 0$$

Describe and correct the error the student made.

4. **Construct Arguments** Yuki says, *"You can check the solution(s) of rational equations in any of the steps of the solution process."* Explain why her reasoning is incorrect.

Do You KNOW HOW?

Solve.

5. $\dfrac{4}{x+6} = 2$

6. $\dfrac{x^2}{x+3} = \dfrac{9}{x+3}$

7. Organizing given information into a table can be helpful when solving rate problems. Use this table to solve the following problem.

	Distance	Rate	Time
Upstream			
Downstream			

The speed of a stream is 4 km/h. A boat can travel 6 km upstream in the same time it takes to travel 12 km downstream. Find the speed of the boat in still water.

River Current = 4 km/h

Upstream: 6 km

Downstream: 12 km
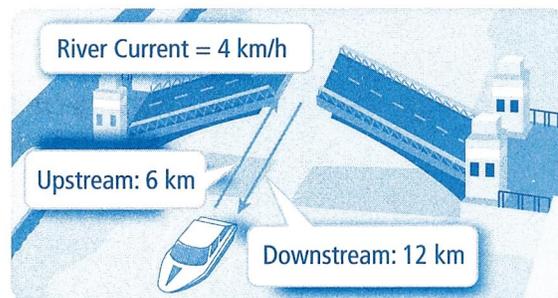
✎ PRACTICE & PROBLEM SOLVING

UNDERSTAND

8. **Reason** If you solve a work-rate problem and your solution, which represents the amount of time it would take *working together*, exceeds the individual *working alone* times that are given, then how do you know your solution is unreasonable? Explain.

9. **Construct Arguments** Explain why a negative solution must be eliminated as an extraneous solution when solving a rational equation for an unknown rate.

10. **Error Analysis** Describe and correct the error Miranda made in solving the rational equation.

$$\frac{1}{x-2} + \frac{x-2}{x+2} = \frac{x-4}{x-2}$$

$$(x+2)(x-2)\left(\frac{1}{x-2} + \frac{x-2}{x+2}\right) = \left(\frac{x-4}{x-2}\right)(x+2)(x-2)$$

$$(x+2)(1) + (x-2)(x-2) = (x+4)(x+2)$$

$$x + 2 + \cancel{x^2} - 4x + 4 = \cancel{x^2} + 6x + 8$$

$$-3x + 6 = 6x + 8$$

$$-2 = 9x;\ \text{or}\ x = -\frac{2}{9} \quad \textbf{✗}$$

11. **Generalize** In addition to identifying extraneous solutions, why else is it important to substitute your solution into the original equation?

12. **Mathematical Connections** Explain how solving rational equations is related to solving linear and quadratic equations.

13. **Higher Order Thinking** Write a rational equation that cannot have 2 or −6 as solutions.

14. **Make Sense and Persevere** Solve the rational equation shown. Explain what is unique about the solution.

$$\frac{x^2 - 7x - 18}{x + 2} = x - 9$$

PRACTICE & PROBLEM SOLVING

PRACTICE

Solve the equation. SEE EXAMPLE 1

15. $\dfrac{1}{x-3} = 10$

16. $\dfrac{15}{x+3} = 3$

17. $\dfrac{12}{x-4} = 9$

18. $\dfrac{5}{3-x} = 1$

Solve the problem. SEE EXAMPLE 2

19. Paige can complete a landscaping job in 6 hours. Malia can complete the same job in 4 hours. Working together, how long would it take them to complete the job?

Malia
4 hours

Paige
6 hours

20. Russel and Aaron can build a shed in 8 hours when working together. Aaron works three times as fast as Russel. How long would it take Russel to build the shed if he were to work alone?

Aaron and Russel 8 hours
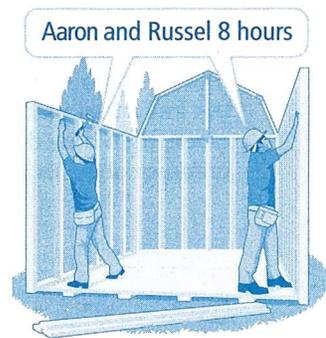
Solve the equation. SEE EXAMPLE 3

21. $\dfrac{x}{x-3} - 4 = \dfrac{3}{x-3}$

22. $\dfrac{x^2}{x-10} = \dfrac{100}{x-10} - 10$

Solve the equation. SEE EXAMPLE 4

23. $\dfrac{4}{3(x+1)} = \dfrac{12}{x^2-1}$

24. $\dfrac{x}{x-3} + \dfrac{2x}{x+3} = \dfrac{18}{(x+3)(x-3)}$

Solve the problem. SEE EXAMPLE 5

25. A boat travels 8 miles upstream in the same amount of time it can travel 12 miles downstream. In still water the speed of the boat is 5 mi/h. What is the speed of the current?

PRACTICE & PROBLEM SOLVING

APPLY

26. Make Sense and Persevere Kenji can finish a puzzle in 2 hours working alone. Oscar can finish the same puzzle in 3 hours working alone. How long would it take Oscar and Kenji to finish the puzzle if they worked on it together?

27. Use Structure A commercial jet flies 1,500 miles with the wind. In the same amount of time it can fly 1,000 miles against the wind. The speed of the jet in still air is 550 mph. Find the speed of the wind.

Wind Direction

1,000 miles
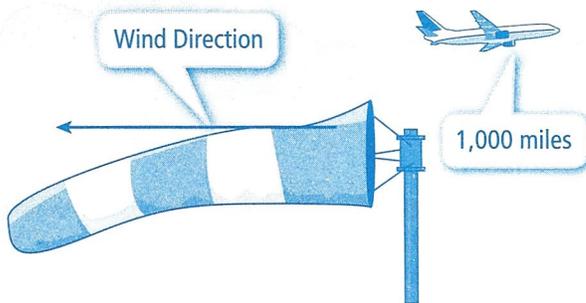
a. Organize the given information and what you need to find in a table.

b. Write and solve a rational equation to find the wind speed.

28. Reason During their day at the beach, Jae and his friends rent a Jet Ski. They split the $120 rental fee evenly among themselves. Then Jae, with only his friend Morgan, share the cost of a $16 pizza. If Jae spends a total of $48 for both, then find the number of friends, n, with whom he shared the cost of the Jet Ski rental.

29. Make Sense and Persevere When driving to their family reunion, River's mom drove 10 miles at a rate of x mph and then 25 miles at a rate of $x + 10$ mph. The total driving time was 45 minutes. What were the two driving speeds at which River's mom drove?

30. Generalize So far this baseball season, Philip has gotten a hit 8 times out of 40 at-bats. He wants to increase his batting average to 0.333. Calculate the number of consecutive hits, h, he would need in order to achieve this goal. Round your answer to the nearest whole number.

☑ **ASSESSMENT PRACTICE**

31. Which of the following rational equations have at least one extraneous solution? Select all that apply.

    Ⓐ $\dfrac{2}{x} = \dfrac{3}{x-4}$

    Ⓑ $\dfrac{x^2}{x-3} = \dfrac{9}{x-3}$

    Ⓒ $\dfrac{x-1}{x-5} = \dfrac{9}{x-5}$

    Ⓓ $x + \dfrac{3}{x} = 4$

    Ⓔ $\dfrac{x}{x-3} - \dfrac{3}{2} = \dfrac{3}{x-3}$

32. **SAT/ACT** Which of the following is the solution of $\dfrac{3}{x+1} = \dfrac{2}{x-3}$?

    Ⓐ $x = -11$

    Ⓑ $x = -\dfrac{7}{5}$

    Ⓒ $x = \dfrac{7}{5}$

    Ⓓ $x = 11$

33. **Performance Task** A chemist needs alcohol solution in the correct concentration for her experiment. She adds a 6% alcohol solution to 50 gallons of solution that is 2% alcohol. The function that represents the percent of alcohol in the resulting solution is $f(x) = \dfrac{50(0.02) + x(0.06)}{50 + x}$, where x is the amount of 6% solution added.

6% alcohol

50 gal
2% alcohol
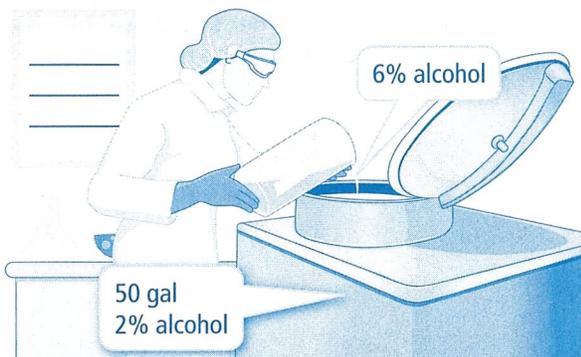
Part A How much 6% solution should be added to create a solution that is 5% alcohol?

Part B Use Appropriate Tools Explain the steps you could take to use your graphing calculator to verify the correctness of your answer to part (A).