



Activity

# 4-4

## Adding and Subtracting Rational Expressions



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### CRITIQUE & EXPLAIN

Teo and Shannon find the following exercise in their homework:

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$$

- A. Teo claims that a common denominator of the sum is  $2 + 3 + 9 = 14$ . Shannon claims that it is  $2 \cdot 3 \cdot 9 = 54$ . Is either student correct? Explain why or why not.

- B. Find the sum, explaining the method you use.

- C. **Construct Arguments** Timothy states that the quickest way to find the sum of any two fractions with unlike denominators is to multiply their denominators to find a common denominator, and then rewrite each fraction with that denominator. Do you agree?

#### HABITS OF MIND

**Look for Relationships** For two fractions with denominators 10 and  $x$ , when is  $10x$  the least common multiple? When is  $10x$  NOT the least common multiple?



### EXAMPLE 1 Try It! Add Rational Expressions With Like Denominators

1. Find the sum.

a.  $\frac{10x-5}{2x+3} + \frac{8-4x}{2x+3}$

b.  $\frac{x-5}{x+5} + \frac{3x-21}{x+5}$

#### HABITS OF MIND

**Make Sense and Persevere** Explain why it does not make sense to add the denominators when adding rational numbers. Use numerical fractions to support your thinking.

### EXAMPLE 2 Try It! Identify the Least Common Multiple of Polynomials

2. Find the LCM for each set of expressions.

a.  $x^3 + 9x^2 + 27x + 27$ ,  $x^2 - 4x - 21$

b.  $10x^2 - 10y^2$ ,  $15x^2 - 30xy + 15y^2$ ,  $x^2 + 3xy + 2y^2$

### EXAMPLE 3 Try It! Add Rational Expressions With Unlike Denominators

3. Find the sum.

a.  $\frac{x+6}{x^2-4} + \frac{2}{x^2-5x+6}$

b.  $\frac{2x}{3x+4} + \frac{4x^2-11x-12}{6x^2+5x-4}$

**EXAMPLE 4** **Try It!** Subtract Rational Expressions

4. Simplify.

a.  $\frac{1}{3x} + \frac{1}{6x} - \frac{1}{x^2}$

b.  $\frac{3x-5}{x^2-25} - \frac{2}{x+5}$

**HABITS OF MIND****Communicate Precisely** How does finding the LCM of two or more polynomials help you to add and subtract rational expressions?**EXAMPLE 5** **Try It!** Find a Rate

5. On the way to work Juan carpools with a fellow co-worker, and then takes the city bus back home in the evening. The average speed of the 20-mi trip is 5 mph faster in the carpool. Write an expression that represents Juan's total travel time.

**HABITS OF MIND****Construct Arguments** Does Juan spend more time in the carpool or riding the bus? How do you know?**EXAMPLE 6** **Try It!** Simplify a Compound Fraction

6. Simplify.

a.  $\frac{\frac{1}{x-1}}{\frac{x+1}{3} + \frac{4}{x-1}}$

b.  $\frac{\frac{2-1}{x}}{\frac{x+2}{x}}$

**HABITS OF MIND****Reason** Edwin multiplied the top and bottom of the fraction in problem 6 part (a) by  $\frac{3}{x+1} + \frac{x-1}{4}$ . Will this technique work to simplify the compound fraction? Explain.

## Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do you rewrite rational expressions to find sums and differences?

2. **Vocabulary** In your own words, define **compound fraction** and provide an example of one.

3. **Error Analysis** A student added the rational expressions as follows:  

$$\frac{5x}{x+7} + \frac{7}{x} = \frac{5x}{x+7} + \frac{7(7)}{x+7} = \frac{5x+49}{x+7}$$
  
 Describe and correct the error the student made.

4. **Construct Arguments** Explain why, when stating the domain of a sum or difference of rational expressions, not only should the simplified sum or difference be considered but the original expression should also be considered.

5. **Make Sense and Persevere** In adding or subtracting rational expressions, why is the L in LCD significant?

## Do You KNOW HOW?

6. Find the sum of  $\frac{3}{x+1} + \frac{11}{x+1}$ .

Find the LCM of the polynomials.

7.  $x^2 - y^2$  and  $x^2 - 2xy + y^2$

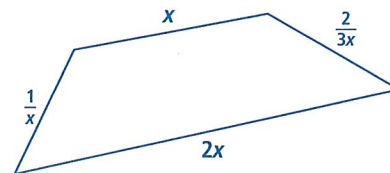
8.  $5x^3y$  and  $15x^2y^2$

Find the sum or difference.

9.  $\frac{3x}{4y^2} - \frac{y}{10x}$

10.  $\frac{9y+2}{3y^2-2y-8} + \frac{7}{3y^2+y-4}$

11. Find the perimeter of the quadrilateral in simplest form.



**PRACTICE & PROBLEM SOLVING****UNDERSTAND**

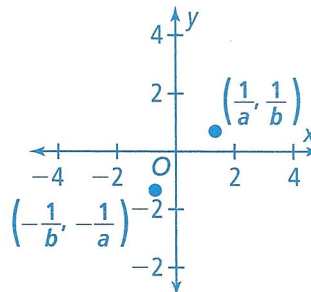
12. **Generalize** Explain how addition and subtraction of rational expressions is similar to and different from addition and subtraction of rational numbers.

13. **Error Analysis** Describe and correct the error a student made in adding the rational expressions

$$\begin{aligned}
 \frac{1}{x^2 + 3x + 2} + \frac{x^2 + 4x}{4x + 8} &= \frac{1}{(x + 1)(x + 2)} + \frac{x(x + 4)}{4(x + 2)} \\
 &= \frac{4}{4(x + 1)(x + 2)} + \frac{x(x + 4)}{4(x + 1)(x + 2)} \\
 &= \frac{4 + x^2 + 4x}{4(x + 1)(x + 2)} \\
 &= \frac{x^2 + 4x + 4}{4(x + 1)(x + 2)} \\
 &= \frac{(x + 2)(x + 2)}{4(x + 1)(x + 2)} \\
 &= \frac{x + 2}{4(x + 1)} \cdot \frac{(x + 2)}{(x + 2)} \\
 &= \frac{x + 2}{4(x + 1)}
 \end{aligned}$$

**X**

14. **Higher Order Thinking** Find the slope of the line that passes through the points shown. Express in simplest form.



15. **Reason** For what values of  $x$  is the sum of  $\frac{x - 5y}{x + y}$  and  $\frac{x + 7y}{x + y}$  undefined? Explain.

16. **Error Analysis** A student says that the LCM of  $3x^2 + 7x + 2$  and  $9x + 3$  is  $(3x^2 + 7x + 2)(9x + 3)$ . Describe and correct the error the student made.



# PRACTICE & PROBLEM SOLVING

## PRACTICE

Find the sum. SEE EXAMPLE 1

17.  $\frac{4x}{x+7} + \frac{9}{x+7}$

18.  $\frac{3y-1}{y^2+4y} + \frac{9y+6}{y(y+4)}$

Find the LCM for each group of expressions.

SEE EXAMPLE 2

19.  $x^2 - 7x + 6, x^2 - 5x - 6$

20.  $y^2 + 2y - 24, y^2 - 16, 2y$

Find the sum. SEE EXAMPLE 3

21.  $\frac{6x}{x^2-8x} + \frac{4}{2x-16}$

22.  $\frac{3y}{2y^2-y} + \frac{2}{2y}$

Find the difference. SEE EXAMPLE 4

23.  $\frac{4x}{x^2-1} - \frac{4}{x-1}$

24.  $\frac{y-1}{3y+15} - \frac{y+3}{5y+25}$

25. On Saturday morning, Ahmed decided to take a bike ride from one end of the 15-mile bike trail to the other end of the bike trail and back. His average speed the first half of the ride was 2 mph faster than his speed on the second half. Find an expression for Ahmed's total travel time. If his average speed for the first half of the ride was 12 mph, how long was Ahmed's bike ride? SEE EXAMPLE 5



Rewrite as a rational expression. SEE EXAMPLE 6

26.  $\frac{1 + \frac{1}{x}}{x - \frac{1}{x}}$

27.  $\frac{\frac{3}{y} + \frac{7}{x}}{\frac{1}{y} - \frac{2}{x}}$

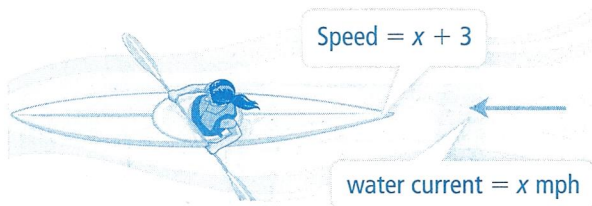
28.  $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{a^2-b^2}{ab}}$

29.  $\frac{\frac{z^2-z-12}{z^2-2z-15}}{\frac{z^2+8z+12}{z^2-5z-14}}$

## PRACTICE & PROBLEM SOLVING

### APPLY

30. **Use Structure** Aisha paddles a kayak 5 miles downstream at a rate 3 mph faster than the river's current. She then travels 4 miles back upstream at a rate 1 mph slower than the river's current. Hint: Let  $x$  represent the rate of the river current.



- Write and simplify an expression to represent the total time it takes Aisha to paddle the kayak 5 miles downstream and 4 miles upstream.
- If the rate of the river current,  $x$ , is 2 mph, how long was Aisha's entire kayak trip?

31. **Model With Mathematics** Rectangles A and B are similar. An expression that represents the width of each rectangle is shown. Find the scale factor of rectangle A to rectangle B in simplest form.



32. **Reason** The Taylor family drives 180 miles (round trip) to a professional basketball game. On the way to the game, their average speed is approximately 8 mph faster than their speed on the return trip home.
- Let  $x$  represent their average speed on the way home. Write and simplify an expression to represent the total time it took them to drive to and from the game.
  - If their average speed going to the game was 72 mph, how long did it take them to drive to the game and back?

# ASSESSMENT PRACTICE

33. Which of the following compound fractions simplifies to  $\frac{x+1}{x-3}$ ? Select all that apply.

(A)  $\frac{\frac{x^2+5x+4}{x^2+2x-8}}{\frac{x^2-4x+3}{x^2-3x+2}}$

(B)  $\frac{\frac{x^2-1}{x^2-4}}{\frac{x^2+x-7}{x^2+5x+6}}$

(C)  $\frac{\frac{x^2+3x-10}{x^2-16}}{\frac{x^2-4x-5}{x^2-1}}$

(D)  $\frac{\frac{x^2+3x-10}{x^2-5x+6}}{\frac{x^2-25}{x^2-4x-5}}$

34. **SAT/ACT** What is the difference between

$\frac{x}{9}$  and  $\frac{x-y}{6}$ ?

(A)  $\frac{5x-y}{18}$

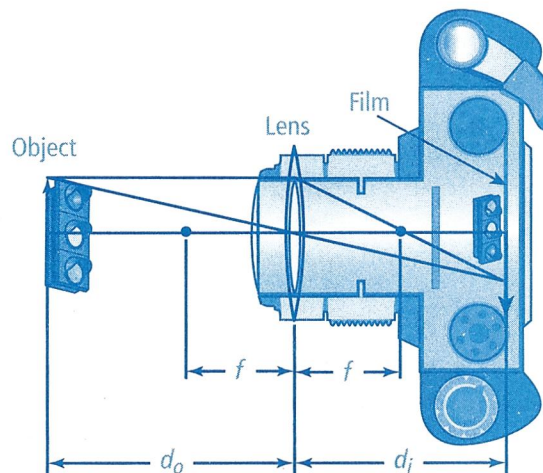
(B)  $\frac{5x+y}{18}$

(C)  $\frac{-x+3y}{18}$

(D)  $\frac{-x-3y}{18}$

35. **Performance Task** The lens equation

$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$  represents the relationship between  $f$ , the focal length of a camera lens,  $d_i$ , the distance from the lens to the film, and  $d_o$ , the distance from the lens to the object.



**Part A** Find the focal length of a camera lens if an object that is 12 cm from a camera lens is in focus on the film when the lens is 6 cm from the film.

**Part B** Suppose the focal length of another camera lens is 3 inches, and the object to be photographed is 5 feet away. What distance (to the nearest tenth inch) should the lens be from the film?