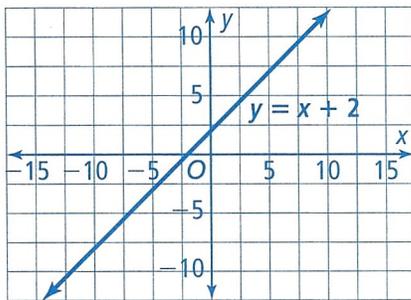




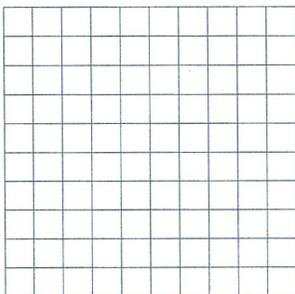
4-3

Multiplying and
Dividing Rational
Expressions**EXPLORE & REASON**

Consider the following graph of the function $y = x + 2$.



- A. What is the domain of this function?
- B. Sketch a function that resembles the graph, but restrict its domain to exclude 2.



- C. **Use Structure** Consider the function you have sketched. What kind of function might have a graph like this? Explain.

HABITS OF MIND

Reason Does the graph of $y = \frac{2x + 6}{x + 3}$ have a vertical asymptote at $x = -3$? Explain.

EXAMPLE 1  **Try It! Write Equivalent Rational Expressions**

1. Write an expression equivalent to $\frac{3x^5 - 18x^4 - 21x^3}{2x^6 - 98x^4}$.
Remember to give the domain for your expression.

EXAMPLE 2  **Try It! Simplify a Rational Expression**

2. Simplify each expression and show the domain for which the identity with the two expressions is valid.

a. $\frac{x^2 + 2x + 1}{x^3 - 2x^2 - 3x}$

b. $\frac{x^3 + 4x^2 - x - 4}{x^2 + 3x - 4}$

HABITS OF MIND

Critique Reasoning Bailey simplified the rational expression $\frac{x^2 + 2x + 4}{x^2 + x + 2}$ by

dividing out the x^2 -terms, and then dividing out a factor of $x + 2$ to get 2 as the simplified form of the rational expression. Is Bailey correct? Why or why not?

EXAMPLE 3  **Try It! Multiply Rational Expressions**

3. Find the simplified form of each product, and give the domain.

a. $\frac{x^2 - 16}{9 - x} \cdot \frac{x^2 + x - 90}{x^2 + 14x + 40}$

b. $\frac{x + 3}{4x} \cdot \frac{3x - 18}{6x + 18} \cdot \frac{x^2}{4x + 12}$

**EXAMPLE 4** **Try It! Multiply a Rational Expression by a Polynomial**

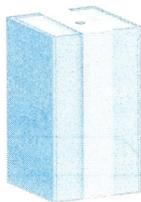
4. Find the simplified form of each product and the domain.

$$\text{a. } \frac{x^3 - 4x}{6x^2 - 13x - 5} \cdot (2x^3 - 3x^2 - 5x) \qquad \text{b. } \frac{3x^2 + 6x}{x^2 - 49} \cdot (x^2 + 9x + 14)$$

HABITS OF MIND**Generalize** Why is it important to identify the domain of a rational expression before you simplify it rather than after?**EXAMPLE 5** **Try It! Divide Rational Expressions**

5. Find the simplified quotient and the domain of each expression.

$$\text{a. } \frac{1}{x^2 + 9x} \div \left(\frac{6 - x}{3x^2 - 18x} \right) \qquad \text{b. } \frac{2x^2 - 12x}{x + 5} \div \left(\frac{x - 6}{x + 5} \right)$$

EXAMPLE 6 **Try It! Use Division of Rational Expressions**6. The company compares the ratios of surface area to volume for two more containers. One is a rectangular prism with a square base. The other is a rectangular prism with a rectangular base. One side of the base is equal to the side-length of the first container, and the other side is twice as long. The surface area of this second container is $4x^2 + 6xh$. The heights of the two containers are equal. Which has the smaller surface area-to-volume ratio?**HABITS OF MIND****Use Structure** Is the domain of the quotient $\frac{2x^2 - 12x}{x + 5} \div \left(\frac{x - 6}{x + 5} \right)$ different from the domain of the product $\left(\frac{2x^2 - 12x}{x + 5} \right) \left(\frac{x - 6}{x + 5} \right)$? Explain.

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How does understanding operations with fractions help you multiply and divide rational expressions?

2. **Vocabulary** In your own words, define rational expression and provide an example of a rational expression.

3. **Error Analysis** A student divided the rational expressions as follows:

$$\frac{4x}{5y} \div \frac{20x^2}{25y^2} = \frac{4x}{5y} \div \frac{4 \cdot 20x^2}{25y^2} = \frac{16x^3}{25y^3}$$

Describe and correct the errors the student made.

4. **Communicate Precisely** Why do you have to state the domain when simplifying rational expressions?

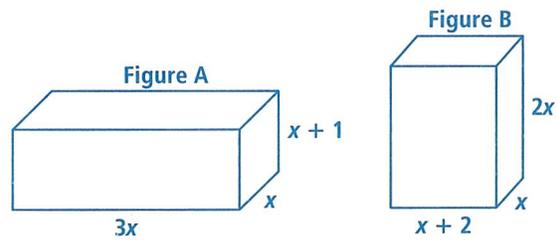
Do You KNOW HOW?

5. What is the simplified form of the rational expression $\frac{x^2 - 36}{x^2 + 3x - 18}$? What is the domain?

6. Find the product and give the domain of

$$\frac{y+3}{y+2} \cdot \frac{y^2+4y+4}{y^2-9}$$

7. Find and simplify the ratio of the volume of Figure A to the volume of Figure B.



PRACTICE & PROBLEM SOLVING

UNDERSTAND

8. **Reason** Explain why $\frac{4x^2-7}{4x^2-7} = 1$ is a valid identity under the domain of all real numbers except $\pm \frac{\sqrt{7}}{2}$.

9. **Error Analysis** Describe the error a student made in multiplying and simplifying

$$\frac{x+2}{x-2} \cdot \frac{x^2-4}{x^2+x-2}$$

$$\begin{aligned} & \frac{x+2}{x-2} \cdot \frac{x^2-4}{x^2+x-2} \\ &= \frac{\cancel{x+2}}{\cancel{x-2}} \cdot \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-1)} \\ &= \frac{2}{-1} \end{aligned}$$

X

10. **Higher Order Thinking** Explain why the process of dividing by a rational number is the same as multiplying by its reciprocal.

11. **Use Appropriate Tools** Explain how you can use your graphing calculator to show that the rational expressions $\frac{-6x^2+21x}{3x}$ and $-2x+7$ are equivalent under a given domain. What is true about the graph at $x=0$ and why?

12. **Generalize** Explain the similarities between rational numbers and rational expressions.

13. **Use Structure** Determine whether $\frac{5x+11}{6x+11} = \frac{5}{6}$ is *sometimes, always, or never* true. Justify your reasoning.

14. **Construct Arguments** Explain how you can tell whether a rational expression is in simplest form.

15. **Communicate Precisely** When multiplying $\frac{15}{x} \cdot \frac{x}{3} = 5$, is it necessary to make the restriction $x \neq 0$? Why or why not?

16. **Reason** If the denominator of a rational expression is $x^3 + 3x^2 - 10x$, what value(s) must be restricted from the domain for x ?



PRACTICE & PROBLEM SOLVING

PRACTICE

Write an equivalent expression. State the domain. SEE EXAMPLE 1

17. $\frac{x^3 + 4x^2 - 12x}{x^2 + x - 30}$

18. $\frac{3x^2 + 15x}{x^2 + 3x - 10}$

What is the simplified form of each rational expression? What is the domain? SEE EXAMPLE 2

19. $\frac{y^2 - 5y - 24}{y^2 + 3y}$

20. $\frac{ab^3 - 9ab}{12ab^2 + 12ab - 144a}$

21. $\frac{x^2 + 8x + 15}{x^2 - x - 12}$

22. $\frac{x^3 + 9x^2 - 10x}{x^3 - 9x^2 - 10x}$

Find the product and the domain. SEE EXAMPLE 3

23. $\frac{x^2 + 6x + 8}{x^2 + 4x + 3} \cdot \frac{x + 3}{x + 2}$

24. $\frac{(x - y)^2}{x + y} \cdot \frac{3x + 3y}{x^2 - y^2}$

Find the product and the domain. SEE EXAMPLE 4

25. $\frac{(x + 5)}{(x^3 - 25x)} \cdot (2x^3 - 11x^2 + 5x)$

26. $\frac{(2x^2 - 10x)}{(x - 5)(x^2 - 1)} \cdot (3x^2 + 4x + 1)$

Find the quotient and the domain. SEE EXAMPLE 5

27. $\frac{y^2 - 16}{y^2 - 10y + 25} \div \frac{3y - 12}{y^2 - 3y - 10}$

28. $\frac{(x - y)^2}{x + y} \div \frac{3x + 3y}{x^2 - y^2}$

29. $\frac{25x^2 - 4}{x^2 - 9} \div \frac{5x - 2}{x + 3}$

30. $\frac{x^4 + x^3 - 30x^2}{x^2 - 3x - 18} \div \frac{x^3 + x^2 - 30x}{x^2 - 36}$

31. A rectangular prism with a volume of $3x^3 + 7x^2 + 2x$ cubic units has a base area of $x^2 + 2x$ square units. Find the height of the rectangular prism. SEE EXAMPLE 6

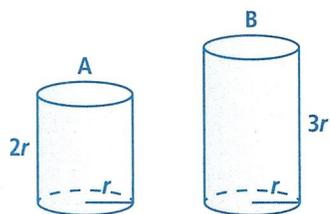
PRACTICE & PROBLEM SOLVING

APPLY

32. **Make Sense and Persevere** An engineering firm wants to construct a cylindrical structure that will maximize the volume for a given surface area. Compare the ratios of the volume to surface area of each of the cylindrical structures shown, using the following formulas for volume and surface area of cylinders.

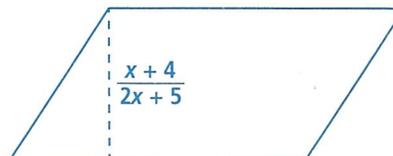
$$\text{Volume } (V) = \pi r^2 h$$

$$\text{Surface Area } (SA) = 2\pi r h + 2\pi r^2$$

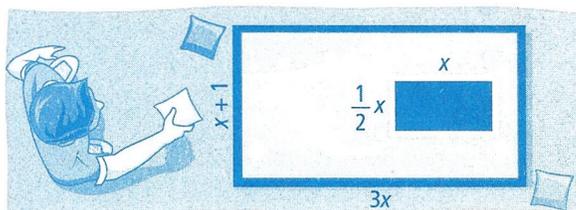


- Calculate the ratio of volume to surface area for cylinder A.
- Calculate the ratio of volume to surface area for cylinder B.
- Which of these cylinders has a greater ratio of volume to surface area?

33. **Look for Relationships** A parallelogram with an area of $\frac{3x + 12}{10x + 25}$ square units has a height shown. Find the length of the base of the parallelogram.



34. **Model With Mathematics** Brie designed a carnival game that involves tossing a beanbag into the box shown. In order to win a prize, the beanbag must fall inside the black rectangle. The probability of winning is equal to the ratio of the area of the black rectangle to the total area of the face of the box shown. Find this probability in simplified form.



ASSESSMENT PRACTICE

35. Which of the following rational expressions simplify to $\frac{y}{y+3}$? Select all that apply.

(A) $\frac{(2y^2 + y)(y + 3)}{(4y + 2)(y + 3)^2}$

(B) $\frac{3y^2 + y}{3y^2 + 10y + 3}$

(C) $\frac{2y^3 + 3y^2 + y}{(2y + 1)(y^2 + 4y + 3)}$

(D) $\frac{y^2 + 2y}{y^2 + 4y + 3}$

(E) $\frac{1}{y + 3}$

36. **SAT/ACT** For what value of x is $\frac{2x^2 + 8x}{(x + 4)(x^2 - 9)}$ undefined?

(A) -8

(B) -3

(C) 0

(D) 4

(E) 9

37. **Performance Task** The approximate annual interest rate r of a monthly installment loan is given by the formula:

$$r = \frac{\left[\frac{24(nm - p)}{n} \right]}{\left(p + \frac{nm}{12} \right)},$$

where n is the total number of payments, m is the monthly payment, and p is the amount financed.

Part A Find the approximate annual interest rate (to the nearest percent) for a four-year signature loan of \$20,000 that has monthly payments of \$500.

Part B Find the approximate annual interest rate (to the nearest tenth percent) for a five-year auto loan of \$40,000 that has monthly payments of \$750.