Activity

4-2 Graphing Rational Functions

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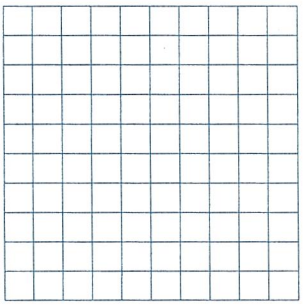
EXPLORE & REASON

Look at the three functions shown.

$$f(x) = x - 1$$
$$g(x) = \frac{x-1}{2}$$
$$h(x) = \frac{x-1}{x-2}$$

A. **Look for Relationships** Graph each function. Determine which of the functions are linear. Find the y-intercept of each function and the slope, if appropriate.



B. What is the effect on the graph of f when dividing $x - 1$ by 2?

C. What happens to the graph of h as x approaches 2?

D. **Communicate Precisely** What is the effect on the graph of $f(x)$ when dividing $x - 1$ by $x - 2$? (Hint: Compare it to what you found in part (b).)

HABITS OF MIND

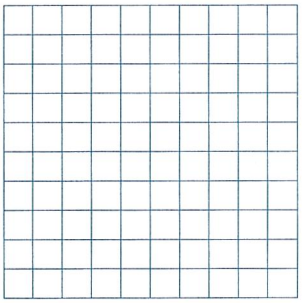
Look for Relationships What similarities do you notice between the graph of $h(x) = \frac{x-1}{x-2}$ and the graph of a reciprocal function?

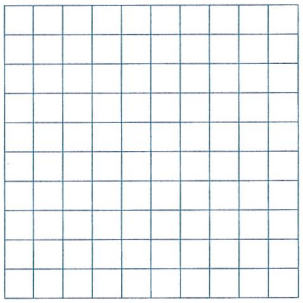
EXAMPLE 1 Try It! Rewrite a Rational Function to Identify Asymptotes

1. Use long division to rewrite each rational function. Find the asymptotes of f and sketch the graph.

   a. $f(x) = \frac{6x}{2x + 1}$

   b. $\frac{x}{x - 6}$





EXAMPLE 2 Try It! Find Asymptotes of a Rational Function

2. What are the vertical and horizontal asymptotes of the graph of each function?

   a. $g(x) = \frac{2x^2 + x - 9}{x^2 - 2x - 8}$

   b. $f(x) = \frac{x^2 + 5x + 4}{3x^2 - 12}$

HABITS OF MIND

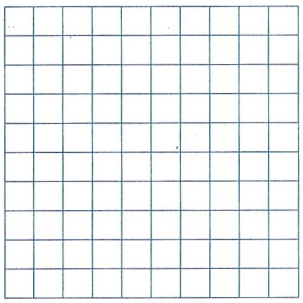
Model With Mathematics Under what conditions could there be a horizontal asymptote at $y = -2$? Give an example.

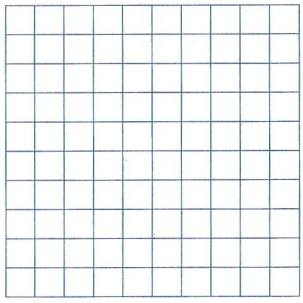
EXAMPLE 3 Try It! Graph a Function of the Form $\frac{ax + b}{cx + d}$

3. Graph each function.

   a. $f(x) = \frac{4x - 3}{x + 8}$

   b. $g(x) = \frac{3x + 2}{x - 1}$





EXAMPLE 4 Try It! Use a Rational Function Model

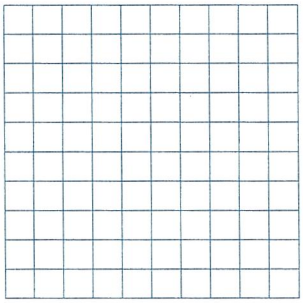
4. New techniques have changed the cost function. For the new function $g(p) = \frac{3.2p + 1}{100 - p}$, what percent of the pollutant can be removed for \$50 million?

HABITS OF MIND

Make Sense and Persevere What are the asymptotes for the function $g(p) = \frac{3.2p + 1}{100 - p}$?

EXAMPLE 5 Try It! Graph a Rational Function

5. Identify the asymptotes and sketch the graph of $g(x) = \frac{x^2 - 5x + 6}{2x^2 - 10}$.



HABITS OF MIND

Reason When will the graph of a rational function have two vertical asymptotes?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you graph a rational function?

2. **Vocabulary** Why does it make sense to call the expressions in this lesson *rational* functions?

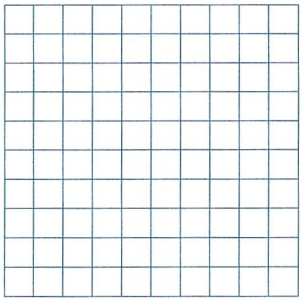
3. **Error Analysis** Ashton said the graph of $f(x) = \frac{x+2}{2x^2+4x-6}$ has a horizontal asymptote at $y = \frac{1}{2}$. Describe and correct Ashton's error.

4. **Reason** When will the graph of a rational function have no vertical asymptotes? Give an example of such a function.

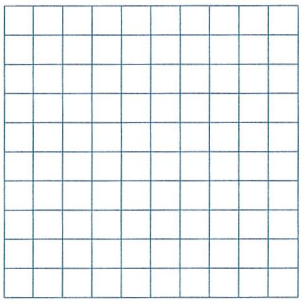
Do You KNOW HOW?

Find the vertical asymptote(s) and horizontal asymptote(s) of the rational function. Then graph the function.

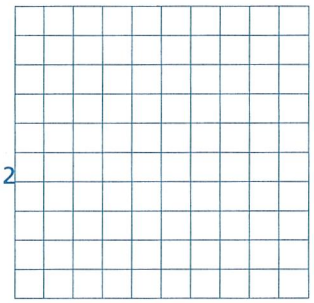
5. $f(x) = \frac{x+2}{x-3}$

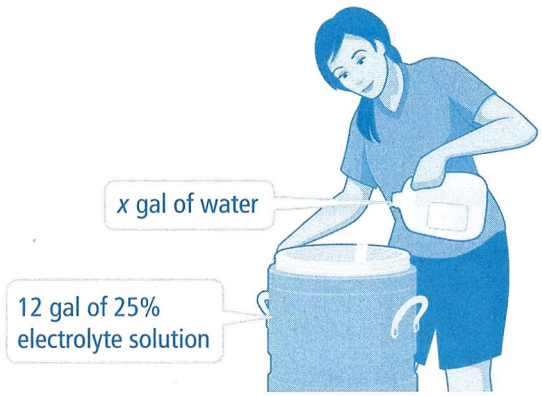


6. $f(x) = \frac{x-1}{2x+1}$



7. A trainer mixed water with an electrolyte solution. The relationship can be modeled by $f(x) = \frac{3}{\quad} x + 12$ the function.





PRACTICE & PROBLEM SOLVING

UNDERSTAND

8. **Communicate Precisely** What is the horizontal asymptote of the rational function $f(x) = \frac{ax^2 + bx + c}{dx^2 + ex + f}$? Explain.

9. **Error Analysis** Juanita is trying to determine the vertical and horizontal asymptotes for the graph of the function $f(x) = \frac{x^2 + 3x - 4}{x^2 - x - 12}$. Describe and correct the error Juanita made in determining the vertical and horizontal asymptotes.

$$f(x) = \frac{x^2 + 3x - 4}{x^2 - x - 12}$$

$$= \frac{(x + 4)(x - 1)}{(x + 3)(x - 4)}$$

vertical asymptote: $x = -3$, $x = 4$
horizontal asymptote: $y = -4$, $y = 1$

10. **Higher Order Thinking** Suppose the numerator and denominator of a rational function are factored, and the numerator and denominator have a common factor of $x + a$. What happens on the graph of the function at $x = -a$? Explain your reasoning.

11. **Reason** The graph of a rational function has vertical asymptotes at $x = -3$ and $x = 1$ and a horizontal asymptote at $y = 3$.

a. Write a function that has these attributes.

b. Graph your function to verify it is correct.

c. Is it possible to have a different graph with the same attributes? Explain.

12. **Communicate Precisely** Explain how to use the end behavior of the function $f(x) = \frac{x^2 + 6}{4x^2 - 3x - 1}$ to determine the horizontal asymptote of the graph. Then explain why using end behavior for finding the horizontal asymptote works the same as using the ratio of the leading terms.

PRACTICE & PROBLEM SOLVING

PRACTICE

Use long division to rewrite each rational function. What are the asymptotes of f? Sketch the graph.
SEE EXAMPLE 1

13. $f(x) = \frac{2x}{x+4}$

14. $f(x) = \frac{5x}{x-2}$

15. $f(x) = \frac{6x^2}{3x^2+1}$

16. $f(x) = \frac{x^2}{2x^2-2}$

Identify the vertical and horizontal asymptotes of each rational function. SEE EXAMPLE 2

17. $f(x) = \frac{3x^2}{4x^2-1}$

18. $f(x) = \frac{5x+6}{x^2-9x+18}$

19. $f(x) = \frac{4x+3}{x^2-4}$

20. $f(x) = \frac{5x^2-19x-4}{2x^2-2}$

Graph each function. SEE EXAMPLE 3

21. $f(x) = \frac{-1}{x+3}$

22. $f(x) = \frac{3x}{x-1}$

23. $f(x) = \frac{x+2}{-x+1}$

24. $f(x) = \frac{2x-3}{3x+4}$

25. An owner tracks her sales each day since opening her marketing company. The daily sales, in dollars, after day x is given by the function $f(x) = \frac{200{,}000x}{x^2+150}$. On approximately which day(s) will the daily sales be \$3,000?
SEE EXAMPLE 4

AC ADVERTISING COMPANY

DAILY SALES TRACKER

DAYS	SALES
1	\$1,324.50
2	\$2,597.40
3	\$3,773.58
4	\$4,819.28

Graph each function, labeling all horizontal or vertical asymptotes of the form $x = a$ or $y = b$.
SEE EXAMPLE 5

26. $f(x) = \frac{x+4}{2x^2-13x-7}$

27. $f(x) = \frac{2x-1}{x^2-3x-10}$

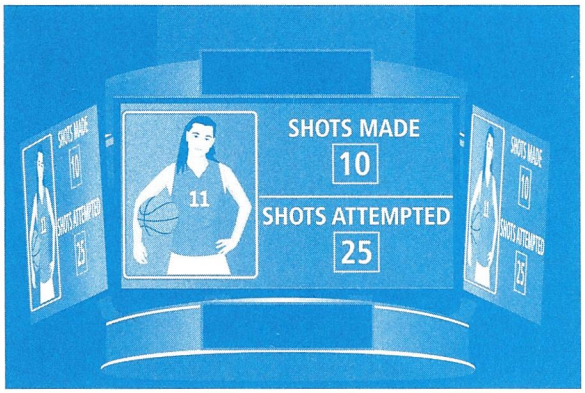
28. $f(x) = \frac{x^2+x-2}{2x^2-9x-18}$

29. $f(x) = \frac{6x^2-12x}{x^2+5x-24}$

PRACTICE & PROBLEM SOLVING

APPLY

30. Make Sense and Persevere Amaya made 10 three-point shots out of 25 attempts. If she then goes on to make x consecutive three-point shots, her success would be given by the function $f(x) = \frac{x + 10}{x + 25}$.



a. Identify the vertical asymptote(s) and horizontal asymptote(s).

b. Graph the function.

31. Model With Mathematics A software CD can be manufactured for \$0.10 each. The development cost to produce the software is \$500,000. The first 200 CDs were used by testers to test the functionality of the software and were not sold.

a. Write a function f for the average cost, in dollars, of a salable software CD where x is the number of salable software CDs.

b. What are the vertical asymptotes of the graph?

c. What are the horizontal asymptotes of the graph?

d. Graph the function.

e. What do the asymptotes mean?

32. Reason After diluting salt water, the concentration of salt in the water is given by the function $f(x) = \frac{0.5x}{x^2 - 1}$, where x is the time in hours since the dilution.

a. What is the concentration of salt in the water after 4 hours?

b. After how many hours will the concentration of salt in the water be 0.2? Round to the nearest hundredth.

ASSESSMENT PRACTICE

33. Which function has a graph with a vertical asymptote at $x = 3$? Select all that apply.

Ⓐ $f(x) = \frac{x - 2}{x^2 + 2x - 15}$

Ⓑ $f(x) = \frac{x - 3}{x^2 + 7x + 12}$

Ⓒ $f(x) = \frac{x^2 - 9}{x + 9}$

Ⓓ $f(x) = \frac{x^2 + 6x + 5}{x^2 - 9}$

34. **SAT/ACT** Which function has a graph with a horizontal asymptote at $y = -1$?

Ⓐ $f(x) = \frac{x + 5}{x - 3}$

Ⓑ $f(x) = \frac{-x + 9}{x - 8}$

Ⓒ $f(x) = \frac{x^2 + 4}{x^2 - 1}$

Ⓓ $f(x) = \frac{2x^2}{x^2 - x - 2}$

35. **Performance Task** There is a relationship between the degree of the numerator and denominator of a rational function and the function's horizontal asymptote.

Function	Horizontal Asymptote
$f(x) = \frac{2x}{x^2}$	
$f(x) = \frac{5x^2}{2x^3}$	
$f(x) = \frac{9x^6}{7x}$	
$f(x) = \frac{-3x^7}{4x^4}$	

Part A Complete the right column of the table.

Part B What is the relationship between the degree of the numerator and denominator when the horizontal asymptote is $y = 0$?

Part C What is the relationship between the degree of the numerator and denominator when there is no a horizontal asymptote?